Chapter 3: Resistive Network Analysis – Instructor Notes

Chapter 3 presents the principal topics in the analysis of resistive (DC) circuits. The presentation of node voltage and mesh current analysis is supported by several solved examples and drill exercises, with emphasis placed on developing consistent solution methods, and on reinforcing the use of a systematic approach. The aim of this style of presentation, which is perhaps more detailed than usual in a textbook written for a non-majors audience, is to develop good habits early on, with the hope that the orderly approach presented in Chapter 3 may facilitate the discussion of AC and transient analysis in Chapters 4 and 5. Make The Connection sidebars (pp. 65-67) introduce analogies between electrical and thermal circuit elements. These analogies are to be encountered again in Chapter 5. A brief discussion of the principle of superposition precedes the discussion of Thévenin and Norton equivalent circuits. Again, the presentation is rich in examples and drill exercises, because the concept of equivalent circuits will be heavily exploited in the analysis of AC and transient circuits in later chapters. The Focus on Methodology boxes (p. 66 – Node Analysis; p. 76 – Mesh Analysis; pp. 93, 97, 101 – Equivalent Circuits) provide the student with a systematic approach to the solution of all basic network analysis problems. Following a brief discussion of maximum power transfer, the chapter closes with a section on nonlinear circuit elements and load-line analysis. This section can be easily skipped in a survey course, and may be picked up later, in conjunction with Chapter 9, if the instructor wishes to devote some attention to load-line analysis of diode circuits. Finally, those instructors who are used to introducing the op-amp as a circuit element, will find that sections 8.1 and 8.2 can be covered together with Chapter 3, and that a good complement of homework problems and exercises devoted to the analysis of the op-amp as a circuit element is provided in Chapter 8. Modularity is a recurrent feature of this book, and we shall draw attention to it throughout these Instructor Notes.

The homework problems present a graded variety of circuit problems. Since the aim of this chapter is to teach solution techniques, there are relatively few problems devoted to applications. We should call the instructor's attention to the following end-of-chapter problems: 3.30 on the Wheatstone bridge; 3.33 and 3.34 on fuses; 3.35-3.37 on electrical power distribution systems; 3.76-83 on various nonlinear resistance devices. The chapter includes 83 problems, as well as 25 fully solved exercises.

Learning Objectives for Chapter 3

1. Compute the solution of circuits containing linear resistors and independent and dependent sources using node analysis.
2. Compute the solution of circuits containing linear resistors and independent and dependent sources using mesh analysis.
3. Apply the principle of superposition to linear circuits containing independent sources.
5. Use equivalent circuits ideas to compute the maximum power transfer between a source and a load.
6. Use the concept of equivalent circuit to determine voltage, current and power for nonlinear loads using load-line analysis and analytical methods.
Sections 3.1, 3.2, 3.3, 3.4: Nodal and Mesh Analysis

Focus on Methodology: Node Voltage Analysis Method

1. Select a reference node (usually ground). This node usually has most elements tied to it. All other nodes will be referenced to this node.
2. Define the remaining \( n-1 \) node voltages as the independent or dependent variables. Each of the \( m \) voltage sources in the circuit will be associated with a dependent variable. If a node is not connected to a voltage source, then its voltage is treated as an independent variable.
3. Apply KCL at each node labeled as an independent variable, expressing each current in terms of the adjacent node voltages.
4. Solve the linear system of \( n-1-m \) unknowns.

Focus on Methodology: Mesh Current Analysis Method

1. Define each mesh current consistently. Unknown mesh currents will be always defined in the clockwise direction; known mesh currents (i.e., when a current source is present) will always be defined in the direction of the current source.
2. In a circuit with \( n \) meshes and \( m \) current sources, \( n-m \) independent equations will result. The unknown mesh currents are the \( n-m \) independent variables.
3. Apply KVL to each mesh containing an unknown mesh current, expressing each voltage in terms of one or more mesh currents.
4. Solve the linear system of \( n-m \) unknowns.

Problem 3.1

Note: the rightmost top resistor missing a value should be 1 \( \Omega \).

Solution:

Known quantities:
Circuit shown in Figure P3.1

Find:
Voltages \( v_1 \) and \( v_2 \).

Analysis:

Applying KCL at each of the two nodes, we obtain the following equations:

\[
\frac{V_1}{3} + \frac{V_1 - V_2}{1} + 4 = 0
\]

\[
\frac{V_2}{2} + \frac{V_2}{2} = \frac{V_2 - V_1}{1} = 0
\]

Rearranging the equations,

\[
\frac{4}{3} V_1 - V_2 = 4
\]

\[-V_1 + 2V_2 = 0
\]

Solving the equations,

\[
V_1 = 4.8 \text{ V and } V_2 = 2.4 \text{ V}
\]
Problem 3.2

Solution:

Known quantities:
Circuit shown in Figure P3.2

Find:
Voltages $v_1$ and $v_2$.

Analysis:
Applying KCL at each node, we obtain:

\[
\frac{v_1 - 20}{30} + \frac{v_1}{20} - \frac{v_2}{10} = 0
\]

\[
\frac{v_2}{30} + \frac{v_2}{30} + \frac{v_2 - v_1}{10} = 0
\]

Rearranging the equations,

\[
5.5v_1 - 3v_2 = 20
\]

\[
-3v_1 + 5v_2 = 0
\]

Solving the two equations,

$v_1 = 5.41 \text{ V}$ and $v_2 = 3.24 \text{ V}$

Problem 3.3

Note: ignore the “floating” arrow pointing up in the top mesh.

Solution:

Known quantities:
Circuit shown in Figure P3.3

Find:
Voltages across the resistance.

Analysis:

At node 1:

\[
\frac{v_1 - v_2}{1} + \frac{v_1 - v_3}{0.5} = 2
\]

At node 2:

\[
\frac{v_2 - v_1}{1} + \frac{v_2}{0.25} = 3
\]

At node 3:

\[
\frac{v_3 - v_1}{0.25} + \frac{v_3}{0.33} = -3
\]

Solving for $v_2$, we find $v_2 = 0.34 \text{ V}$ and, therefore, $v = 0.34 \text{ V}$.
Problem 3.4

Solution:

Known quantities:
Circuit shown in Figure P3.4

Find:
Current through the voltage source.

Analysis:

At node 1:
\[ \frac{v_1 - v_2}{0.5} + \frac{v_1 - v_3}{0.5} = -2 \]  (1)

At node 2:
\[ \frac{v_2 - v_1}{0.5} + \frac{v_2}{0.25} + i = 0 \]  (2)

At node 3:
\[ \frac{v_3 - v_1}{0.5} + \frac{v_3}{0.33} - i = 0 \]  (3)

Further, we know that \( v_3 = v_2 + 3 \). Now we can eliminate either \( v_2 \) or \( v_3 \) from the equations, and be left with three equations in three unknowns:

\[ \frac{v_1 - v_2}{0.5} + \frac{v_1 - (v_2 + 3)}{0.5} = -2 \]  (1)

\[ \frac{v_2 - v_1}{0.5} + \frac{v_2}{0.25} + i = 0 \]  (2)

\[ \frac{(v_2 + 3) - v_1}{0.5} + \frac{(v_2 + 3)}{0.33} - i = 0 \]  (3)

Solving the three equations we compute

\[ i = 8.286 \text{A} \]

Problem 3.5

Solution:

Known quantities:
Circuit shown in Figure P3.5 with mesh currents: \( I_1 = 5 \text{A} \), \( I_2 = 3 \text{A} \), \( I_3 = 7 \text{A} \).

Find:
The branch currents through:

a) \( R_1 \),
b) \( R_2 \),
c) \( R_3 \).

Analysis:

a) Assume a direction for the current through \( R_1 \) (e.g., from node \( A \) to node \( B \)). Then summing currents at node \( A \):

\[ KCL: \quad -I_1 + I_{R1} + I_3 = 0 \]

\[ I_{R1} = I_1 - I_3 = -2 \text{A} \]
This can also be done by inspection noting that the assumed direction of the current through $R_1$ and the direction of $I_1$ are the same.

b) Assume a direction for the current through $R_2$ (e.g., from node B to node A). Then summing currents at node B:

$$KCL: \quad I_2 + I_{R2} - I_3 = 0$$

$$I_{R2} = I_3 - I_2 = 4 \text{ A}$$

This can also be done by inspection noting that the assumed direction of the current through $R_2$ and the direction of $I_3$ are the same.

c) Only one mesh current flows through $R_3$. If the current through $R_3$ is assumed to flow in the same direction, then:

$$I_{R1} = I_3 = 7 \text{ A}.$$  

---

**Problem 3.6**

**Solution:**

**Known quantities:**
Circuit shown in Figure P3.5 with source and node voltages: $V_{S1} = V_{S2} = 110 \text{ V, } V_A = 103 \text{ V, } V_B = -107 \text{ V.}$

**Find:**
The voltage across each of the five resistors.

**Analysis:**
Assume a polarity for the voltages across $R_1$ and $R_2$ (e.g., from ground to node A, and from node B to ground). $R_1$ is connected between node A and ground; therefore, the voltage across $R_1$ is equal to this node voltage. $R_2$ is connected between node B and ground; therefore, the voltage across $R_2$ is equal to the negative of this voltage.

$$V_{R1} = V_A = 103 \text{ V, } V_{R2} = -V_B = +107 \text{ V}$$

The two node voltages are with respect to the ground which is given.

Assume a polarity for the voltage across $R_3$ (e.g., from node B to node A). Then:

$$KVL: \quad V_A + V_{R3} + V_B = 0$$

$$V_{R3} = V_A - V_B = 210 \text{ V}$$

Assume polarities for the voltages across $R_4$ and $R_5$ (e.g., from node A to ground , and from ground to node B):

$$KVL: \quad -V_{S1} + V_{R4} + V_A = 0 \quad KVL: \quad -V_{S2} - V_B - V_{R5} = 0$$

$$V_{R4} = V_{S1} - V_A = 7 \text{ V} \quad V_{R5} = -V_{S2} - V_B = -3 \text{ V}$$
Problem 3.7

Solution:

Known quantities:
Circuit shown in Figure P3.7 with known source currents and resistances, \( R_1 = 3 \ \Omega, R_2 = 1 \ \Omega, R_3 = 6 \ \Omega \).

Find:
The currents \( I_1, I_2 \) using node voltage analysis.

Analysis:
At node 1:
\[
\left( \frac{v_1 - v_2}{3} \right) + \frac{v_2}{1} = 1
\]
At node 2:
\[
\frac{v_2 - v_1}{1} + \frac{v_2}{6} = -2
\]
Solving, we find that:
\[
v_1 = -1.5 \ \text{V} \quad \text{Then,} \quad i_1 = \frac{v_1}{3} = -0.5 \ \text{A}
\]
\[
v_2 = -3 \ \text{V} \quad \text{Then,} \quad i_2 = \frac{v_2}{6} = -0.5 \ \text{A}
\]

Problem 3.8

Solution:

Known quantities:
Circuit shown in Figure P3.7 with known source currents and resistances, \( R_1 = 3\Omega, R_2 = 1\Omega, R_3 = 6\Omega \).

Find:
The currents \( I_1, I_2 \) using mesh analysis.

Analysis:
At mesh (a):
\[
i_a = 1 \ \text{A}
\]
At mesh (b):
\[
3 \left( i_b - i_a \right) + i_b + 6 \left( i_b - i_c \right) = 0
\]
At mesh (c):
\[
i_c = 2 \ \text{A}
\]
Solving, we find that:
\[
i_b = 1.5 \ \text{A}
\]
Then,
\[
i_1 = \left( i_a - i_b \right) = -0.5 \ \text{A}
\]
\[
i_2 = \left( i_b - i_c \right) = -0.5 \ \text{A}
\]
Problem 3.9

Solution:

Known quantities:
Circuit shown in Figure P3.9 with resistance values, current and voltage source values.

Find:
The current, $i$, through the voltage source using node voltage analysis.

Analysis:

At node 1:
\[
\frac{v_1}{200} + \frac{v_1 - v_2}{5} + \frac{v_1 - v_2}{100} = 0
\]

At node 2:
\[
\frac{v_2 - v_1}{5} + i + 0.2 = 0
\]

At node 3:
\[
- i + \frac{v_3 - v_1}{100} + \frac{v_3}{50} = 0
\]

For the voltage source we have:
\[v_3 - v_2 = 50 \, \text{V}\]

Solving the system, we obtain:
\[v_1 = -45.53 \, \text{V}, \quad v_2 = -48.69 \, \text{V}, \quad v_3 = 1.31 \, \text{V}\]
and, finally, $i = 491 \, \text{mA}$.

Problem 3.10

Solution:

Known quantities:
The current source value, the voltage source value and the resistance values for the circuit shown in Figure P3.10.

Find:
The three node voltages indicated in Figure P3.10 using node voltage analysis.

Analysis:

At node 1:
\[
\frac{v_1}{200} + \frac{v_1 - v_2}{75} = 0.2 \, \text{A}
\]

At node 2:
\[
\frac{v_2 - v_1}{75} + \frac{v_2}{25} + \frac{v_2 - v_3}{50} + i = 0
\]

At node 3:
\[
- i + \frac{v_3 - v_2}{50} + \frac{v_3}{100} = 0
\]

For the voltage source we have: $v_3 + 10 = v_2$

Solving the system, we obtain:
\[v_1 = 14.24 \, \text{V}, \quad v_2 = 4.58 \, \text{V}, \quad v_3 = -5.42 \, \text{V}\]
and, finally, $i = -254 \, \text{mA}$.
Problem 3.11

Solution:

Known quantities:
The voltage source value, 3 V, and the five resistance values, indicated in Figure P3.11.

Find:
The current, \( i \), drawn from the independent voltage source using node voltage analysis.

Analysis:

At node 1:
\[
\frac{v_1 - 3}{0.5} + \frac{v_1 - v_2}{0.25} = 0
\]

At node 2:
\[
\frac{v_2 - v_1 + v_2}{0.25} + \frac{0.5}{0.75} = 0
\]

Solving the system, we obtain:
\( v_1 = 1.125 \, \text{V} \), \( v_2 = 0.75 \, \text{V} \)

Therefore, \( i = \frac{3 - v_1}{0.5} = 3.75 \, \text{A} \).

Problem 3.12

Solution:

Known quantities:
The circuit shown in Figure P3.12.

Find:
Power delivered to the load resistance.

Analysis:

KCL at node 1:
\[
\frac{V_1}{R_I} + \frac{V_1 - V_2}{R_V} = 0.5
\]

Or
\( 3 \, V_1 - V_2 = 6 \) (Eq. 1)

KCL at node 2:
\[
\frac{V_1 - V_2}{R_V} = \frac{V_2}{R_I || (R_2 + R_L)}
\]

Or
\( 14 \, V_2 - 4 \, V_1 = -16 \) (Eq. 2)

substitute Eq. 1 into Eq. 2
\( V_2 = -0.6316 \)

and by voltage divider:
\[
V_L = \left( \frac{R_L}{R_2 + R_L} \right) V_2 = -0.316 \, \text{V}
\]

\[
P_L = \frac{V_L^2}{R_L} = 25 \, \text{mW}
\]
**Problem 3.13**

**Solution:**

**Known quantities:**
Circuit shown in Figure P3.13.

**Find:**

a) Voltages
b) Write down the equations in matrix form.

**Analysis:**

a) Using conductances, apply KCL at node 1:
\[(G_1 + G_{12} + G_{13})V_1 - G_{12}V_2 - G_{13}V_3 = I_s\]

Then apply KCL at node 2:
\[-G_{12}V_1 + (G_2 + G_{12} + G_{23})V_2 - G_{23}V_3 = 0\]

and at node 3:
\[-G_{13}V_1 - G_{23}V_2 + (G_3 + G_{13} + G_{23})V_3 = 0\]

Rewriting in the form

\[[G][V] = [I]\]

we have

\[
\begin{bmatrix}
G_1 + G_{12} + G_{13} & -G_{12} & -G_{13} \\
-G_{12} & G_2 + G_{12} + G_{23} & -G_{23} \\
-G_{13} & -G_{23} & G_3 + G_{13} + G_{23}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
= \begin{bmatrix}
I_s \\
0 \\
0
\end{bmatrix}
\]

b) The result is identical to that obtained in part a).

**Problem 3.14**

**Solution:**

Circuit shown in Figure P3.14.

**Find:**
Current $i_1$ and $i_2$.

**Analysis:**

For mesh #1:

\[i_1 (1 + 3) + i_2 (-3) = 1\]

For mesh #2:

\[i_1 (-3) + i_2 (3 + 2) = -2\]

Solving,

\[i_1 = -0.091A\]
\[i_2 = -0.455A\]
Problem 3.15

Solution:
Circuit shown in Figure P3.15.

Find:
Current $i_1$ and $i_2$ and voltage across the resistance $10\Omega$.

Analysis:
Mesh #1 $\left(20 + 15 + 10\right)i_1 - 10i_2 = 0$
Mesh #2 $(10 + 40 + 10)i_2 - 10i_1 = 50$

Therefore, $I_1 = 0.1923$ A and $I_2 = 0.865$ A,
$v_{10\Omega} = 10(I_2 - i_1) = 6.727$ V

Problem 3.16

Solution:
Circuit shown in Figure P3.16.

Find:
Voltage across the $3\Omega$ resistance.

Analysis:
Meshes 1, 2 and 3 are clockwise from the left
For mesh #1:
$i_1(1 + 2 + 3) + i_2(-2) + i_3(-3) = 2$

For mesh #2:
$i_1(-2) + i_2(2 + 2 + 1) + i_3(-1) = -1$

For mesh #3:
$i_1(-3) + i_2(-1) + i_3(3 + 1 + 1) = 0$

Solving,
$i_1 = 0.5224$ A
$i_2 = 0.0746$ A
$i_3 = 0.3284$ A

and $v = 3(i_1 - i_3) = 3(0.194) = 0.5821$ V
Problem 3.17

Note: the right-most mesh current should be labeled $I_3$, not $I_2$.

Solution:

Mesh #1 (on the left-hand side)
\[ 2 - 2I_1 - 3(I_1 - I_2) = 0 \]

If we treat mesh #2 (middle) and mesh #3 (on the right-hand side) as a single loop containing the four resistors (but not the current source), we can write
\[ -I_2 - 3I_3 - 2I_3 - 3(I_2 - I_1) = 0 \]

From the current source:
\[ I_3 - I_2 = 2 \]

Solving the system of equations:
\[ I_1 = -0.333 \text{ A} \quad I_2 = -1.222 \text{ A} \quad I_3 = 0.778 \text{ A} \]

Problem 3.18

Solution:

Circuit shown in Figure P3.18.

Find:
Voltage across the current source.

Analysis:

Meshes 1, 2 and 3 go from left to right.

For mesh #1:
\[ i_1(2 + 3) + i_2(-3) + i_3(0) = 2 \]

For meshes #2 and #3:
\[ i_1(-3) + i_2(1 + 3) + i_3(3 + 2) = 0 \]

For the current source:
\[ i_1(0) + i_2(0) + i_3(-1) = -2 \]

Solving,
\[ i_3 = 0.778 \text{ A} \]
and \[ v = i_3(3 + 2) = 3.89 \text{ V} \]
Problem 3.19

Solution:
Circuit shown in Figure P3.19.
Find:
Mesh equation in matrix form.
Analysis:
\[
\begin{bmatrix}
R_{12} + R_{13} & -R_{12} & -R_{13} \\
-R_{12} & R_{12} + R_2 + R_{23} & -R_{23} \\
-R_{13} & -R_{23} & R_2 + R_{13} + R_{23}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
= \begin{bmatrix}
V_S \\
0 \\
0
\end{bmatrix}
\]

b) same result as a).

Problem 3.20

Solution:
Circuit shown in Figure P3.20.
Find:
Mesh equation in matrix form and solve for currents.
Analysis:
after source transformation, we can have the equivalent circuit shown in the right hand side. We can write down the following matrix
\[
\begin{bmatrix}
6 + 4 + 4 & -4 & -4 \\
-4 & 4 + 4 + 8 & 0 \\
-4 & 0 & 2 + 4
\end{bmatrix}
\begin{bmatrix}
I_{1,2} \\
I_3 \\
I_4
\end{bmatrix}
= \begin{bmatrix}
12 - 3 \\
3 \\
5
\end{bmatrix}
\]
Solve the equation, we can have
\[I_{1,2} = 1.2661 \text{A} = I_2\]
\[I_3 = 0.5040 \text{A}\]
\[I_4 = 1.6774 \text{A}\]
\[I_1 = 2 \text{A}\]
Problem 3.21

Solution:

Known quantities:

Circuit of Figure P3.21 with voltage source, \( V_S \), current source, \( I_S \), and all resistances.

Find:

a. The node equations required to determine the node voltages.

b. The matrix solution for each node voltage in terms of the known parameters.

Analysis:

a) Specify the nodes (e.g., \( A \) on the upper left corner of the circuit in Figure P3.10, and \( B \) on the right corner). Choose one node as the reference or ground node. If possible, ground one of the sources in the circuit. Note that this is possible here. When using KCL, assume all unknown current flow out of the node. The direction of the current supplied by the current source is specified and must flow into node A.

\[
-KCL:\quad -I_S + \frac{V_a - V_b}{R_2} + \frac{V_a - V_b}{R_1} = 0
\]

\[
\begin{align*}
KCL: & \quad V_a \left( \frac{1}{R_2} + \frac{1}{R_1} \right) + V_b \left( \frac{1}{R_1} \right) = I_S + \frac{V_S}{R_2} \\
& \quad \frac{V_b - V_a}{R_1} + \frac{V_b - V_S}{R_3} + \frac{V_b - 0}{R_4} = 0
\end{align*}
\]

b) Matrix solution:

\[
\begin{align*}
V_a &= \begin{bmatrix}
\frac{I_S + V_S}{R_2} & -\frac{1}{R_1} \\
\frac{V_S}{R_3} & \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \\
\frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} \\
\frac{1}{R_1} & \frac{1}{R_3} + \frac{1}{R_4}
\end{bmatrix} \\
& \quad \begin{bmatrix}
I_S + V_S \\
\frac{1}{R_3} \\
\frac{1}{R_1} + \frac{1}{R_2}
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
V_b &= \begin{bmatrix}
\frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} \\
\frac{1}{R_1} & \frac{1}{R_3} + \frac{1}{R_4} \\
\frac{1}{R_1} & \frac{1}{R_3} + \frac{1}{R_4}
\end{bmatrix} \\
& \quad \begin{bmatrix}
\frac{1}{R_1} \\
\frac{1}{R_3} \\
\frac{1}{R_1} + \frac{1}{R_2}
\end{bmatrix}
\end{align*}
\]

Notes:

1. The denominators are the same for both solutions.
2. The main diagonal of a matrix is the one that goes to the right and down.
3. The denominator matrix is the "conductance" matrix and has certain properties:
   a) The elements on the main diagonal \([i](row) = j(column)\] include all the conductance connected to node \(i=j\).
   b) The off-diagonal elements are all negative.
   c) The off-diagonal elements are all symmetric, i.e., the \(i j\)-th element = \(j i\)-th element. This is true only because there are no controlled (dependent) sources in this circuit.
   d) The off-diagonal elements include all the conductance connected between node \(i\) [row] and node \(j\) [column].
Problem 3.22

Solution:

Known quantities:
Circuit shown in Figure P3.22
\( V_{S1} = V_{S2} = 110 \text{ V} \)
\( R_1 = 500 \text{ m}\Omega \quad R_2 = 167 \text{ m}\Omega \)
\( R_3 = 700 \text{ m}\Omega \)
\( R_4 = 200 \text{ m}\Omega \quad R_5 = 333 \text{ m}\Omega \)

Find:

a. The most efficient way to solve for the voltage across \( R_3 \). Prove your case.
b. The voltage across \( R_3 \).

Analysis:

a) There are 3 meshes and 3 mesh currents requiring the solution of 3 simultaneous equations. Only one of these mesh currents is required to determine, using Ohm’s Law, the voltage across \( R_3 \).

In the terminal (or node) between the two voltage sources is made the ground (or reference) node, then three node voltages are known (the ground or reference voltage and the two source voltages). This leaves only two unknown node voltages (the voltages across \( R_1, V_{R1} \), and across \( R_2, V_{R2} \)). Both these voltages are required to determine, using KVL, the voltage across \( R_3, V_{R3} \).

A difficult choice. Choose node analysis due to the smaller number of unknowns. Specify the nodes. Choose one node as the ground node. In KCL, assume unknown currents flow out.

b)

\[
\begin{align*}
KCL: & \quad \frac{V_{R1} - V_{S1}}{R_4} + \frac{V_{R1} - V_{R2}}{R_3} = 0 \\
& \quad \frac{V_{R2} - (-V_{S2})}{R_2} + \frac{V_{R2} - V_{R1}}{R_3} = 0 \\
& \quad \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \quad \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} = 8.43 \text{ \Omega}^{-1} \\
& \quad \frac{1}{R_3} \quad \frac{1}{R_5} \quad 1.43 \text{ \Omega}^{-1} \\
& \quad \frac{1}{R_4} \quad \frac{1}{R_5} \quad 550 \text{ A} \\
& \quad \frac{1}{R_5} \quad \frac{1}{R_5} \quad 330 \text{ A} \\
& \quad \begin{bmatrix} 550 & -1.43 \\ -330 & 8.43 \\ 10.42 & -1.43 \\ \end{bmatrix} = \begin{bmatrix} 5731 & -472 \\ 87.84 & -2.04 \\ \end{bmatrix} = 61.30 \text{ V} \\
& \quad \begin{bmatrix} 8.43 & -1.43 \\ & 10.42 \\ & \end{bmatrix} = \begin{bmatrix} 8.429 & -330 \\ & 550 \\ & \end{bmatrix} = -23.26 \text{ V} \\
& \quad \begin{bmatrix} 85.790 & 85.80 \\ & \end{bmatrix} = V_{R1} - V_{R2} = 84.59 \text{ V} \\
& \quad \begin{bmatrix} -V_{R1} + V_{R3} + V_{R2} = 0 \\ V_{R3} = V_{R1} - V_{R2} = 84.59 \text{ V} \\ \end{bmatrix}
\end{align*}
\]

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. Limited distribution permitted only to teachers and educators for course preparation. If you are a student using this Manual, you are using it without permission.
Problem 3.23

Solution:

Known quantities:
Circuit shown in Figure P3.23

\[ V_{S2} = kT \quad k = 10 \text{ V/°C} \]
\[ V_{S1} = 24 \text{ V} \quad R_S = R_1 = 12 \text{ kΩ} \]
\[ R_2 = 3 \text{ kΩ} \quad R_3 = 10 \text{ kΩ} \]
\[ R_4 = 24 \text{ kΩ} \quad V_{R3} = -2.524 \text{ V} \]

The voltage across \( R_3 \), which is given, indicates the temperature.

Find:

The temperature, \( T \).

Analysis:

Specify nodes (A between \( R_1 \) and \( R_3 \), C between \( R_3 \) and \( R_2 \)) and polarities of voltages (\( V_A \) from ground to A, \( V_C \) from ground to C, and \( V_{R3} \) from C to A). When using KCL, assume unknown currents flow out.

KVL:

\[-V_A + V_{R3} + V_C = 0\]
\[V_C = V_A - V_{R3}\]

Now write KCL at node C, substitute for \( V_C \), solve for \( V_A \):

KCL:

\[\frac{V_C - V_{S1}}{R_2} + \frac{V_C - V_A}{R_3} + \frac{V_C}{R_4} = 0\]
\[\frac{V_A}{R_3} + \left( V_A - V_{R3} \right) \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) = 0\]
\[V_A = \frac{V_{S1}}{R_2} + V_{R3} \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) = \frac{24}{3 \cdot 10^3} + \left( -2.524 \right) \left( \frac{1}{3 \cdot 10^3} + \frac{1}{10 \cdot 10^3} + \frac{1}{24 \cdot 10^3} \right) = 18.14 \text{ V}\]
\[V_C = V_A - V_{R3} = 18.14 - (-2.524) = 20.66 \text{ V}\]

Now write KCL at node A and solve for \( V_{S2} \) and \( T \):

KCL:

\[\frac{V_A - V_{S1}}{R_1} + \frac{V_A - V_{S2}}{R_S} + \frac{V_A - V_C}{R_3} = 0\]
\[V_{S2} = V_A + \frac{R_S}{R_1} \left( V_A - V_{S1} \right) + \frac{R_S}{R_3} \left( V_A - V_C \right) = 18.14 + \frac{12 \cdot 10^3}{12 \cdot 10^3} (18.14 - 24) + \frac{12 \cdot 10^3}{10 \cdot 10^3} (18.14 - 20.66) = 9.26 \text{ V}\]
\[T = \frac{V_{S2}}{k} = \frac{9.26}{10} = 0.926 \text{ °C}\]
Problem 3.24

Solution:

Known quantities:
Circuit shown in Figure P3.24
\( V_S = 5 \text{ V} \quad A_V = 70 \quad R_1 = 2.2 \text{ k}\Omega \)
\( R_2 = 1.8 \text{ k}\Omega \quad R_3 = 6.8 \text{ k}\Omega \quad R_4 = 220 \text{ } \Omega \)

Find:
The voltage across \( R_4 \) using KCL and node voltage analysis.

Analysis:
Node analysis is not a method of choice because the dependent source is [1] a voltage source and [2] a floating source. Both factors cause difficulties in a node analysis. A ground is specified. There are three unknown node voltages, one of which is the voltage across \( R_4 \). The dependent source will introduce two additional unknowns, the current through the source and the controlling voltage (across \( R_1 \)) that is not a node voltage. Therefore 5 equations are required:

\[
\begin{align*}
\text{[1] } & \frac{1}{R_1} V_1 - \frac{V_S}{R_1} + \frac{V_1 - V_3}{R_3} + \frac{V_1 - V_2}{R_2} = 0 \\
\text{[2] } & \frac{1}{R_2} V_2 - \frac{V_S}{R_2} + \frac{I_{CS}}{R_2} = 0 \\
\text{[3] } & \frac{1}{R_3} V_3 + \frac{I_{CS} + V_3}{R_3} = 0 \\
\text{[4] } & \frac{1}{R_4} V_4 = 0 \\
\text{[5] } & V_S - A_V V_{R1} + V_2 = 0 \quad V_2 = V_3 + A_V V_{R1} = V_3 + A_V (V_S - V_1)
\end{align*}
\]

Substitute using Equation [5] into Equations [1], [2] and [3] and eliminate \( V_2 \) (because it only appears twice in these equations). Collect terms:

\[
\begin{align*}
V_1 \left( \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2} + \frac{A_V}{R_2} \right) + V_3 \left( \frac{1}{R_3} + \frac{1}{R_4} \right) + I_{CS} = \frac{V_S}{R_1} + \frac{V_S A_V}{R_2} \\
V_1 \left( -\frac{1}{R_2} + \frac{A_V}{R_2} \right) + V_3 \left( \frac{1}{R_2} \right) + I_{CS} = -\frac{V_S A_V}{R_2} \\
V_1 \left( -\frac{1}{R_3} \right) + V_3 \left( \frac{1}{R_4} \right) + I_{CS} = 0
\end{align*}
\]

Notes:
1. This solution was not difficult in terms of theory, but was terribly long and arithmetically cumbersome. This was because the wrong method was used. There are only 2 mesh currents in the circuit; the sources were voltage sources; therefore, a mesh analysis is the method of choice.
2. In general, a node analysis will have fewer unknowns (because one node is the ground or reference node) and will, in such cases, be preferable.
Problem 3.25

Solution:

Known quantities:
The values of the resistors and of the voltage sources (see Figure P3.25).

Find:
The voltage across the 10 Ω resistor in the circuit of Figure P3.25 using mesh current analysis.

Analysis:

For mesh (a):
\[ i_a (50 + 20 + 20) - i_b (20) - i_c (20) = 12 \]

For mesh (b):
\[ -i_a (20) + i_b (20 + 10) - i_c (10) + 5 = 0 \]

For mesh (c):
\[ -i_a (20) - i_b (10) + i_c (20 + 10 + 15) = 0 \]

Solving,
\[ i_a = 127.5 \text{ mA} \]
\[ i_b = -67.8 \text{ mA} \]
\[ i_c = 41.6 \text{ mA} \]
and \[ v_{R_a} = 10 \left( i_b - i_c \right) = 10 \left( -0.109 \right) = -1.09 \text{ V} \].

Problem 3.26

Solution:

Known quantities:
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.26.

Find:
The voltage across the current source using mesh current analysis.

Analysis:

For mesh (a):
\[ i_a (20 + 30) + i_b (-30) = 3 \]

For meshes (b) and (c):
\[ i_a (-30) + i_b (10 + 30) + i_c (30 + 20) = 0 \]

For the current source: \( i_c - i_b = 0.5 \)

Solving,
\[ i_a = -133 \text{ mA}, \quad i_b = -322 \text{ mA} \text{ and } i_c = 178 \text{ mA} \].

Therefore,
\[ v = i_c (30 + 20) = 8.89 \text{ V} \].

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. Limited distribution permitted only to teachers and educators for course preparation. If you are a student using this Manual, you are using it without permission.
Problem 3.27

Solution:

Known quantities:
The values of the resistors and of the voltage source in the circuit of Figure P3.27.

Find:
The current $i$ through the resistance $R_4$ using mesh current analysis.

Analysis:
For mesh (a):

$$i_a(50 + 1200) + i_b(-1200) = 5.6$$

For meshes (b) and (c):

$$i_a(-1200) + i_b(1200 + 330) + i_c(440) = 0$$

For the current source:

$$i_c - i_b = 0.2V_x = 0.2 \left(1200 \left(i_a - i_b\right)\right) = 240 \left(i_a - i_b\right)$$

Solving,

$$i_a = 136 \text{ mA}, \quad i_b = 137 \text{ mA} \text{ and } i_c = -106 \text{ mA}.$$ 

Therefore,

$$i = i_c = -106 \text{ mA}.$$ 

Problem 3.28

Solution:

Known quantities:
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.9.

Find:
The current through the voltage source using mesh current analysis.

Analysis:
For mesh (a):

$$i_a(100 + 5) + i_b(-5) + 50 = 0$$

For the current source:

$$i_b - i_c = 0.2$$

For meshes (b) and (c):

$$-i_a(5) + i_b(200 + 5) + i_c(50) = 50$$

Solving,

$$i_a = -465 \text{ mA}, \quad i_b = 226 \text{ mA} \text{ and } i_c = 26 \text{ mA}.$$ 

Therefore,

$$i = i_c - i_a = 491 \text{ mA}.$$ 

3.18

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. Limited distribution permitted only to teachers and educators for course preparation. If you are a student using this Manual, you are using it without permission.
Problem 3.29

Solution:

Known quantities:
The values of the resistors and of the current source in the circuit of Figure P3.10.

Find:
The current through the voltage source in the circuit of Figure P3.10 using mesh current analysis.

Analysis:

For mesh (a):
\[ i_a (100) + 10 = 0 \]

For mesh (b):
\[ i_b (200 + 75 + 25) + i_c (-25) + 0.2 (-200) = 0 \]

For mesh (c):
\[ i_b (-25) + i_c (50 + 25) = 10 \]

Solving,
\[ i_a = -100 \text{ mA}, \quad i_b = 148 \text{ mA} \text{ and } i_c = 183 \text{ mA} \]

Therefore,
\[ i = i_c - i_a = 283 \text{ mA} \]

Problem 3.30

Solution:

Known quantities:
The values of the resistors in the circuit of Figure P3.30.

Find:
The current in the circuit of Figure P3.30 using mesh current analysis.

Analysis:

Since \( I \) is unknown, the problem will be solved in terms of this current.

For mesh #1, it is obvious that:
\[ i_1 = I \]

For mesh #2:
\[ i_1 (-1) + i_2 \left(1 + \frac{1}{2} + \frac{1}{5}\right) + i_3 \left(-\frac{1}{5}\right) = 0 \]

For mesh #3:
\[ i_1 \left(-\frac{1}{4}\right) + i_2 \left(-\frac{1}{5}\right) + i_3 \left(\frac{1}{4} + \frac{1}{3} + \frac{1}{5}\right) = 0 \]

Solving,
\[ i_2 = 0.645I \]
\[ i_3 = 0.483I \]

Then,
\[ i = i_3 - i_2 \quad \text{and} \quad i = 0.483I - 0.645I = -0.163I \]
Problem 3.31

Solution:

Known quantities:
The values of the resistors of the circuit in Figure P3.31.

Find:
The voltage gain, \( A_V = \frac{v_2}{v_1} \), in the circuit of Figure P3.31 using mesh current analysis.

Analysis:

Note that \( v = \frac{i_1 - i_2}{2} \)

For mesh #1:
\[
i_1 \left(1 + \frac{1}{2}\right) + i_2 \left(-\frac{1}{2}\right) + i_3(0) = v_1
\]

For mesh #2:
\[
i_1 \left(-\frac{1}{2}\right) + i_2 \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{4}\right) + i_3 \left(-\frac{1}{4}\right) = 2v
\]

or
\[
i_1(−1.5) + i_2(2) + i_3(−0.25) = 0
\]

For mesh #3:
\[
i_1(0) + i_2 \left(-\frac{1}{4}\right) + i_3 \left(-\frac{1}{4} + \frac{1}{4}\right) = -2v
\]

or
\[
i_1(1) + i_2(−1.25) + i_3(0.5) = 0
\]

Solving, \( i_3 = -0.16v_1 \)

from which \( v_2 = \frac{1}{4}i_3 = -0.04v_1 \)

and \( A_V = \frac{v_2}{v_1} = -0.04 \)
Problem 3.32

Solution:

Known quantities:
Circuit in Figure P3.21 and the values of the voltage sources, \( V_{S1} = V_{S2} = 450 \) V, and the values of the 5 resistors:
\[
R_1 = 8 \, \Omega \quad R_2 = 5 \, \Omega \\
R_4 = R_5 = 0.25 \, \Omega \quad R_3 = 32 \, \Omega 
\]

Find:
The voltages across \( R_1 \), \( R_2 \) and \( R_3 \) using KCL and node analysis.

Analysis:
Choose a ground/reference node. The node common to the two voltage sources is the best choice. Specify polarity of voltages and direction of the currents.

\[
KCL: \quad \frac{V_{R1} - V_{S1}}{R_4} + \frac{V_{R1} - 0}{R_1} + \frac{V_{R1} - V_{R2}}{R_3} = 0 \\
KCL: \quad \frac{V_{R2} - (-V_{S2})}{R_5} + \frac{V_{R2} - 0}{R_2} + \frac{V_{R2} - V_{R1}}{R_3} = 0
\]

Collect terms in terms of the unknown node voltages:
\[
V_{R1} \left( \frac{1}{R_4} + \frac{1}{R_1} + \frac{1}{R_3} \right) + V_{R2} \left( \frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{V_{S1}}{R_4} \\
V_{R1} \left( -\frac{1}{R_3} \right) + V_{R2} \left( \frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} \right) = -\frac{V_{S2}}{R_5}
\]

Evaluate the coefficients of the unknown node voltages:
\[
\frac{V_{S1}}{R_4} = \frac{V_{S2}}{R_5} = 450 \quad 1.8 \, kA \quad \frac{1}{R_4} = \frac{1}{R_5} = 0.03125 \, \Omega^{-1} \\
\frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_3} = \frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} = 4.14 \, \Omega^{-1} \\
\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} = 4.23 \, \Omega^{-1}
\]

\[
V_{R1} = \begin{bmatrix} 1800 & -4.23 \cdot 10^{-3} \\ -4.16 & 1800 \end{bmatrix} = 429.5 \, V \\
V_{R2} = \begin{bmatrix} 4.156 & 1800 \\ -31.25 \cdot 10^{-3} & 4.23 \end{bmatrix} = -422.2 \, V \\
KVL: \quad -V_{R1} + V_{R3} + V_{R2} = 0 \\
V_{R3} = V_{R1} - V_{R2} = 852.0 \, V
**Problem 3.33**

**Solution:**

**Known quantities:**
Circuit in Figure P3.33 with the values of the voltage sources, \( V_{S1} = V_{S2} = 115 \) V, and the values of the 5 resistors:
- \( R_1 = R_2 = 5 \) Ω
- \( R_3 = 10 \) Ω
- \( R_4 = R_5 = 200 \) mΩ

**Find:**
The new voltages across \( R_1 \), \( R_2 \) and \( R_3 \), in case \( F_1 \) "blows" or opens using KCL and node analysis.

**Analysis:**
Specify polarity of voltages. The ground is already specified. The current through the fuse \( F_1 \) is zero.

\[
KCL: \quad 0 + \frac{V_{R1}}{R_1} + \frac{V_{R2}}{R_3} = 0 \\
KCL: \quad \frac{V_{R2} - (V_{S2})}{R_5} + \frac{V_{R2} - 0}{R_2} + \frac{V_{R2} - V_{R1}}{R_3} = 0
\]

Collect terms in unknown node voltages:
\[
V_{R1} \left( \frac{1}{R_1} + \frac{1}{R_3} \right) + V_{R2} \left( \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{V_{S2}}{R_5}
\]

\[
\frac{1}{R_3} = \frac{1}{10} = 0.1 \text{ Ω}^{-1} \quad \frac{1}{R_3} = \frac{1}{R_3} = 0.3 \text{ Ω}^{-1}
\]

\[
\frac{V_{S2}}{R_5} = \frac{115}{200 \cdot 10^{-3}} = 575 \text{ A} \quad \frac{1}{R_3} + \frac{1}{R_3} = 5.3 \text{ Ω}^{-1}
\]

\[
V_{R1} = \begin{bmatrix} 0 & -0.1 \\ 0.3 & -0.1 \\ -0.1 & 5.3 \\ 0.3 & 0 \end{bmatrix} \begin{bmatrix} (0) - (57.5) \\ (159) - (0.01) \end{bmatrix} = -36.39 \text{ V}
\]

\[
V_{R2} = \begin{bmatrix} 0 & -0.1 \\ 0.3 & 0 \end{bmatrix} \begin{bmatrix} -172.5 \quad (0) \\ 1.58 \quad 1.58 \end{bmatrix} = -109.2 \text{ V}
\]

\[
KVL: \quad -V_{R1} + V_{R3} + V_{R2} = 0 \\
V_{R3} = V_{R1} - V_{R2} = 72.81 \text{ V}
\]

\[
KVL: \quad -V_{S1} + V_{R4} + V_F + V_{R1} = 0 \quad V_{R4} = I_R R_4 = 0 \\
V_F = 115 - 0 - (-36.39) = 151.4 \text{ V}
\]

Note the voltages are strongly dependent on the loads \( (R_1, R_2 \text{ and } R_3) \) connected at the time the fuse blows. With other loads, the result will be quite different.
**Problem 3.34**

**Solution:**

**Known quantities:**
Circuit in Figure P3.33 and the values of the voltage sources, $V_{S1} = V_{S2} = 120$ V, and the values of the 5 resistors:
- $R_1 = R_2 = 2 \, \Omega$
- $R_3 = 8 \, \Omega$
- $R_4 = R_5 = 250 \, \text{m}\Omega$

**Find:**
The voltages across $R_1$, $R_2$, $R_3$, and $F_1$ in case $F_1$ "blows" or opens using KCL and node analysis.

**Analysis:**
Specify polarity of voltages. The ground is already specified. The current through the fuse $F_1$ is zero.

\[ KCL: \quad 0 + \frac{V_{R1} - 0}{R_1} + \frac{V_{R1} - V_{R2}}{R_3} = 0 \]

\[ KCL: \quad \frac{V_{R2} - (-V_{S2})}{R_5} + \frac{V_{R2} - 0}{R_2} + \frac{V_{R2} - V_{R1}}{R_3} = 0 \]

\[
V_{R1}\left(\frac{1}{R_1} + \frac{1}{R_3}\right) + V_{R2}\left(-\frac{1}{R_3}\right) = 0
\]

\[
V_{R1}\left(-\frac{1}{R_3}\right) + V_{R2}\left(\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V_{S2}}{R_5}
\]

\[
\frac{1}{R_3} = \frac{1}{8} = 0.125 \, \text{Ω}^{-1} \quad \frac{1}{R_5} + \frac{1}{R_3} = 0.625 \, \text{Ω}^{-1}
\]

\[
\frac{V_{S2}}{R_5} = \frac{120}{250 \cdot 10^{-5}} = 480 \text{ A} \quad \frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} = 4.625 \, \text{Ω}^{-1}
\]

\[
V_{R1} = \begin{bmatrix} 0 & -0.125 \\ -0.125 & 4.625 \end{bmatrix} \begin{bmatrix} (0) - (60) \\ (2.89) - (0.016) \end{bmatrix} = -20.87 \text{ V}
\]

\[
V_{R2} = \begin{bmatrix} 0.625 & -480 \\ -480 & 2.87 \end{bmatrix} \begin{bmatrix} (-300) - (0) \\ 2.87 \end{bmatrix} = -104.35 \text{ V}
\]

\[ KVL: \quad -V_{R1} + V_{R3} + V_{R2} = 0 \]

\[
V_{R3} = V_{R1} - V_{R2} = 83.48 \text{ V}
\]

\[ KVL: \quad -V_{S1} + V_{R4} + V_F + V_{R1} = 0 \quad V_{R4} = I_1 R_4 = 0 \]

\[
V_F = 120 - 0 - (-20.87) = 140.9 \text{ V}
\]
Problem 3.35

Solution:

Known quantities:
The values of the voltage sources, \(V_{S1} = V_{S2} = V_{S3} = 170\) V, and the values of the 6 resistors in the circuit of Figure P3.35:
\[ R_{W1} = R_{W2} = R_{W3} = 0.7\ \Omega \\
R_1 = 1.9\ \Omega \\
R_2 = 2.3\ \Omega \\
R_3 = 11\ \Omega \\

Find:
a. The number of unknown node voltages and mesh currents.
b. Unknown node voltages.

Analysis:
a) If the node common to the three sources is chosen as the ground/reference node, and the series resistances are combined into single equivalent resistances, there is only one unknown node voltage. On the other hand, there are two unknown mesh currents.
b) A node analysis is the method of choice! Specify polarity of voltages and direction of currents.

\[
R_{eq1} = R_{W1} + R_1 = 2.6\ \Omega \\
R_{eq2} = R_{W2} + R_2 = 3.0\ \Omega \\
R_{eq3} = R_{W3} + R_3 = 11.7\ \Omega \\

KCL:
\[
\frac{v_N - V_{S1}}{R_{eq1}} + \frac{v_N - (-V_{S2})}{R_{eq2}} + \frac{v_N - (-V_{S3})}{R_{eq3}} = 0 \\
\]
\[
\frac{V_{S1} - V_{S2}}{R_{eq1} + R_{eq2}} + \frac{V_{S3}}{R_{eq3}} = \frac{170}{2.6} + \frac{170}{3.0} + \frac{170}{11.7} \\
\]
\[
v_N = \frac{170}{2.6} + \frac{170}{3.0} + \frac{170}{11.7} = -7.234 \text{ V} \\
\]

KVL:
\[
-V_{S1} + I_1 R_{W1} + I_1 R_1 + v_N = 0 \\
I_1 = \frac{V_{S1} - V_N}{R_{W1} + R_1} = \frac{170 - (-7.234)}{2.6} = 68.167 \text{ A} \\
\]

Problem 3.36

Solution:

Known quantities:
The values of the voltage sources, \(V_{S1} = V_{S2} = V_{S3} = 170\) V, the common node voltage, \(V_N = 28.94\) V, and the values of the 6 resistors in the circuit of Figure P3.35:
\[ R_{W1} = R_{W2} = R_{W3} = 0.7\ \Omega \\
R_1 = 1.9\ \Omega \\
R_2 = 2.3\ \Omega \\
R_3 = 11\ \Omega \\

Find:
The current through and voltage across \(R_1\).

Analysis:

\[ KVL: \quad -V_{S1} + I_1 R_{W1} + I_1 R_1 + V_N = 0 \quad I_1 = \frac{V_{S1} - V_N}{R_{W1} + R_1} = \frac{170 - 28.94}{2.6} = 54.26 \text{ A} \]

\[ OL: \quad V_{R1} = I_1 R_1 = (54.26)(1.9) = 103.1 \text{ V} \]
Problem 3.37

Solution:

Known quantities:
The values of the voltage sources, \( V_{S1} = V_{S2} = V_{S3} = 170 \) V, and the values of the 6 resistors in the circuit of Figure P3.35:

\[
R_{W1} = R_{W2} = R_{W3} = 0.7 \, \Omega \\
R_1 = 1.9 \, \Omega \\
R_2 = 2.3 \, \Omega \\
R_3 = 11 \, \Omega
\]

Find:
The mesh (or loop) equations and any additional equation required to determine the current through \( R_1 \) in the circuit shown in Figure P3.24.

Analysis:

\[
\begin{align*}
KVL: & \quad -V_{S1} + I_1 R_{W1} + I_1 R_1 + (I_1 - I_2) R_2 + (I_1 - I_2) R_{W2} - V_{S2} = 0 \\
KVL: & \quad V_{S2} + (I_2 - I_1) R_{W2} + (I_2 - I_1) R_2 + I_2 R_3 + I_2 R_{W3} + V_{S3} = 0 \\
\end{align*}
\]

\[
I_1(R_1 + R_{W1} + R_2 + R_{W2}) + I_2(-R_2 - R_{W2}) = V_{S1} + V_{S2} \\
I_1(-R_2 - R_{W2}) + I_3(R_2 + R_{W2} + R_3 + R_{W3}) = -V_{S2} - V_{S3}
\]

\[
I_{R1} = I_1 \left[ \begin{array}{cc}
R_{S1} + V_{S2} & -(R_2 + R_{W2}) \\
-(V_{S2} + V_{S3}) & (R_2 + R_{W2} + R_3 + R_{W3}) \\
\end{array} \right] \\
= \left[ \begin{array}{cc}
R_{S1} + R_{W1} + R_2 + R_{W2} & -(R_2 + R_{W2}) \\
-(R_2 + R_{W2}) & (R_2 + R_{W2} + R_3 + R_{W3}) \\
\end{array} \right]^{-1} \\

Hence, the assumed polarity of the second and third branch currents is actually reversed.

Problem 3.38

Solution:

Known quantities:
The values of the voltage sources, \( V_{S1} = 90 \) V, \( V_{S2} = V_{S3} = 110 \) V, and the values of the 6 resistors in the circuit of Figure P3.35:

\[
R_{W1} = R_{W2} = R_{W3} = 1.3 \, \Omega \\
R_1 = 7.9 \, \Omega \\
R_2 = R_3 = 3.7 \, \Omega
\]

Find:
The branch currents, using KVL and loop analysis.

Analysis:

Three equations are required. Voltages will be summed around the 2 loops that are meshes, and KCL at the common node between the resistances. Assume directions of the branch currents and the associated polarities of the voltages. After like terms are collected:

\[
\begin{align*}
KVL: & \quad -V_{S1} + I_1 R_{W1} + I_1 R_1 + (I_1 - I_2) R_2 + (I_1 - I_2) R_{W2} - V_{S2} = 0 \\
KVL: & \quad V_{S2} + (I_2 - I_1) R_{W2} + (I_2 - I_1) R_2 + I_2 R_3 + I_2 R_{W3} + V_{S3} = 0 \\
KCL: & \quad I_3 = I_1 + I_2
\end{align*}
\]

Plugging in the given parameters results in the following system of equations:

\[
\begin{align*}
14.2I_1 - 5.0I_2 & = 200 \\
9.2I_1 - 10.0I_2 & = 220 \\
I_3 & = I_1 + I_2
\end{align*}
\]

Solving the system of equations gives:

\[
I_1 = 9.375A, \quad I_2 = -13.375A, \quad I_3 = -4.000A
\]

Hence, the assumed polarity of the second and third branch currents is actually reversed.
Problem 3.39

Solution:

Known quantities:
The value of $V_{S1}, V_{S2}, R_1, R_2, R_3, R_4, R_5$.

Find:
Using KVL and mesh analyze the voltage across $R_1, R_2, R_3$ under normal conditions.

Analysis:

a) KVL:

At node 1: $\frac{V_{S1} - V_1}{R_4} = \frac{V_1 - V_2}{R_1} + \frac{V_1 - V_3}{R_3}$ (1)

At node 2: $V_2 = 0$ (2)

At node 3: $\frac{V_3 + V_{S2}}{R_5} = \frac{V_3}{R_2} + \frac{V_1 - V_3}{R_3}$ (3)

Combine (1), (2), (3), we have $V_1 = 106.5$ V, $V_2 = 0$ V, $V_3 = -106.5$ V

So the voltage across $R_3$ is $106.5 - (-106.5) = 213$ V.

b) Mesh:

Mesh 1(left up): $V_{S1} - R_4 i_1 - R_1 (i_1 - i_3) = 0$ (1)

Mesh 2(left down): $V_{S2} - R_2 (i_2 - i_3) - R_5 i_2 = 0$ (2)

Mesh 3(right): $-R_3 i_3 - R_2 (i_3 - i_2) - R_1 (i_3 - i_1) = 0$ (3)

Combine (1), (2), (3),

We have: $i_1 = 42.6$ A, $i_2 = 42.6$ A, $i_3 = 21.3$ A,

So the voltage across $R_1$ is $0 + (i_1 - i_3) \cdot R_1 = 106.5$ V

the voltage across $R_2$ is $0 - (i_2 - i_3) \cdot R_2 = -106.5$ V

the voltage across $R_3$ is $i_3 \cdot R_3 = 213$ V
Section 3.5: Superposition

Problem 3.40

Solution:

Known quantities:
The values of the voltage sources, \( V_{S1} = 110 \, \text{V} \), \( V_{S2} = 90 \, \text{V} \)
and the values of the 3 resistors in the circuit of Figure P3.40:
\( R_1 = 560 \, \Omega \) \hspace{1em} \( R_2 = 3.5 \, \text{k}\Omega \) \hspace{1em} \( R_3 = 810 \, \Omega \)

Find:
The current through \( R_1 \) due only to the source \( V_{S2} \).

Analysis:
Suppress \( V_{S1} \). Redraw the circuit. Specify polarity of \( V_{R1} \). Choose ground.

\[
KCL: \quad \frac{-V_{R1-2} - 0}{R_1} + \frac{-V_{R1-2} - 0}{R_2} + \frac{-V_{R1-2} - (-V_{S2})}{R_3} = 0
\]

\[
V_{R1-2} = \frac{\frac{V_{S2}}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{90}{\frac{1}{560} + \frac{1}{3500}} = 33.61 \, \text{V}
\]

\[
OL: \quad I_{R1-2} = \frac{V_{R1-2}}{R_1} = \frac{33.61}{560} = 60.02 \, \text{mA}
\]
Problem 3.42

Solution:

Known quantities:
The values of the voltage sources and of the resistors in the circuit of Figure P3.42:

- \( V_{S1} = V_{S2} = 12 \) V
- \( R_1 = R_2 = R_3 = 1 \) k\( \Omega \)

Find:
The voltage across \( R_2 \) using superposition.

Analysis:

Specify the polarity of the voltage across \( R_2 \). Suppress the voltage source \( V_{S1} \) by replacing it with a short circuit.

Redraw the circuit.

\[
R_{eq} = R_1 \parallel R_3 = \frac{1}{2} \text{k}\Omega = 0.5 \text{k}\Omega
\]

\[
V_{R2-2} = \frac{V_{S2}}{R_2 + R_{eq}} = \frac{12 \text{V}(1000)}{1000 + 500} = 8 \text{V}
\]

Suppress the voltage source \( V_{S2} \) by replacing it with a short circuit. Redraw the circuit.

\[
R_{eq} = R_2 \parallel R_3 = \frac{1}{2} \text{k}\Omega = 0.5 \text{k}\Omega
\]

\[
V_{R2-1} = -\frac{V_{S1}}{R_1 + R_{eq}} = \frac{12 \text{V}(0.5 \text{k}\Omega)}{1 \text{k}\Omega + 0.5 \text{k}\Omega} = -4 \text{V}
\]

\[
V_{R2} = V_{R2-1} + V_{R2-2} = -4 \text{V} + 8 \text{V} = 4 \text{V}
\]

Note: Although superposition is necessary to solve some circuits, it is a very inefficient and very cumbersome way to solve a circuit. This method should, if at all possible, be avoided. It must be used when the sources in a circuit are AC sources with different frequencies, or where some sources are DC and others are AC.
Problem 3.43

Solution:

Known quantities:
The values of the voltage sources and of the resistors in the circuit of Figure P3.43:

\[ V_{S1} = V_{S2} = 450 \text{ V} \]
\[ R_1 = 7 \Omega \quad R_2 = 5 \Omega \quad R_3 = 10 \Omega \quad R_4 = R_5 = 1 \Omega \]

Find:
The component of the current through \( R_3 \) that is due to \( V_{S2} \), using superposition.

Analysis:
Suppress \( V_{S1} \) by replacing it with a short circuit. Redraw the circuit. A solution using equivalent resistances looks reasonable. \( R_1 \) and \( R_4 \) are in parallel:

\[ R_{14} = \frac{R_1 R_4}{R_1 + R_4} = \frac{(7)(1)}{7 + 1} = 0.875 \Omega \]

\( R_{14} \) is in series with \( R_3 \):

\[ R_{143} = R_{14} + R_3 = 0.875 + 10 = 10.875 \Omega \]

\[ R_{eq} = R_5 + \left( R_2 \parallel R_{143} \right) = R_5 + \frac{R_2 R_{143}}{R_2 + R_{143}} = R_5 + \frac{(5)(10.875)}{5 + 10.875} = 4.425 \Omega \]

\( OL: \)

\[ I_S = \frac{V_{S2}}{R_{eq}} = \frac{450}{4.425} = 101.695 \text{ A} \]

\( CD: \)

\[ I_{R3-2} = \frac{I_S R_2}{R_2 + R_{143}} = \frac{(101.695)(5)}{5 + 10.875} = 32.03 \text{ A} \]
Problem 3.44

Solution:

**Known quantities:**
The values of the voltage sources and of the resistors in the circuit of Figure P3.35:

\[ V_{S1} = V_{S2} = V_{S3} = 170 \, \text{V} \]
\[ R_{W1} = R_{W2} = R_{W3} = 0.7 \, \Omega \]
\[ R_1 = 1.9 \, \Omega \quad R_2 = 2.3 \, \Omega \quad R_3 = 11 \, \Omega \]

**Find:**
The current through \( R_1 \), using superposition.

**Analysis:**

\[ R_{eq1} = R_{W1} + R_1 = 2.6 \, \Omega \quad R_{eq2} = R_{W2} + R_2 = 3 \, \Omega \quad R_{eq3} = R_{W3} + R_3 = 11.7 \, \Omega \]

Specify the direction of \( I_1 \). Suppress \( V_{S2} \) and \( V_{S3} \). Redraw circuit.

\[ R_{eq} = R_{eq1} + \frac{R_{eq2} R_{eq3}}{R_{eq2} + R_{eq3}} = 4.99 \, \Omega \]

\[ I_{1-1} = \frac{V_{S1}}{R_{eq}} = 34.08 \, \text{A} \]

Suppress \( V_{S1} \) and \( V_{S3} \). Redraw circuit.

\[ \text{KCL: } \frac{V_A - (-V_{S2})}{R_{eq2}} + \frac{V_A}{R_{eq1}} + \frac{V_A}{R_{eq3}} = 0 \]

\[ V_A = -\frac{V_{S2}}{1 + \frac{R_{eq2}}{R_{eq1}} + \frac{R_{eq2}}{R_{eq3}}} = -70.54 \, \text{V} \]

\[ I_{1-2} = \frac{V_A}{R_{eq1}} = 27.13 \, \text{A} \]

Suppress \( V_{S1} \) and \( V_{S2} \). Redraw circuit.

\[ \text{KCL: } \frac{V_A - (-V_{S3})}{R_{eq3}} + \frac{V_A - 0}{R_{eq1}} + \frac{V_A - 0}{R_{eq2}} = 0 \]

\[ V_A = -\frac{V_{S3}}{1 + \frac{R_{eq3}}{R_{eq1}} + \frac{R_{eq3}}{R_{eq2}}} = 18.09 \, \text{V} \]

\[ I_{1-3} = \frac{V_A}{R_{eq1}} = -6.96 \, \text{A} \]

\[ I = I_{1-1} + I_{1-2} + I_{1-3} = 54.25 \, \text{A} \]

Note: Superposition should be used only for special conditions, as stated in the solution to Problem 3.42. In the problem above a better method is:

a. mesh analysis using KVL (2 unknowns)

b. node analysis using KCL (1 unknown but current must be obtained using OL).
Problem 3.45

Solution:

Known quantities:
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.9.

Find:
The current through the voltage source using superposition.

Analysis:
(1) Suppress voltage source $V$. Redraw the circuit.
For mesh (a): $i_a (100 + 5) + i_b (-5) = 0$

For the current source: $i_b - i_c = 0.2$
For meshes (b) and (c): $-i_a (5) + i_b (200 + 5) + i_c (50) = 0$

Solving,

$i_a = 2 \text{ mA}$, $i_b = 39 \text{ mA}$ and $i_c = -161 \text{ mA}$.

Therefore, $i_1 = i_c - i_a = -163 \text{ mA}$.

(2) Suppress current source $I$. Redraw the circuit.
For mesh (a): $i_a (100 + 5) + i_b (-5) + 50 = 0$

For mesh (b): $-i_a (5) + i_b (200 + 5 + 50) = 50$

Solving,

$i_a = -467 \text{ mA}$ and $i_b = 187 \text{ mA}$.

Therefore,

$i_2 = i_b - i_a = 654 \text{ mA}$.

Using the principle of superposition, $i = i_1 + i_2 = 491 \text{ mA}$.

Problem 3.46

Solution:

Known quantities:
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.6.

Find:
The current through the voltage source using superposition.

Analysis:
(1) Suppress voltage source $V$. Redraw the circuit.
For mesh (a): $i_a = 0$

For mesh (b): $i_b (200 + 75 + 25) + i_c (-25) - 40 = 0$
For mesh (c): $i_b (-25) + i_c (25 + 100) = 0$

Solving,

$i_b = 136 \text{ mA}$ and $i_c = 27 \text{ mA}$.

Therefore, $i_1 = i_c = 27 \text{ mA}$.
(2) Suppress current source $I$. Redraw the circuit.
For mesh (a): \[ i_a (50) - 10 = 0 \]
For mesh (b): \[ i_b (200 + 75 + 25) + i_c (-25) = 0 \]
For mesh (c): \[ i_b (-25) + i_c (25 + 100) = -10 \]
Solving,
\[ i_a = 200 \text{ mA}, \quad i_b = -6.8 \text{ mA} \] and \[ i_c = -81 \text{ mA}. \]
Therefore,
\[ i_2 = i_c - i_a = -281 \text{ mA}. \]
Using the principle of superposition, \[ I = i_1 + i_2 = -254 \text{ mA} \]

---

**Problem 3.47**

**Solution:**

**Known quantities:**
The voltage source value, 3 V, and the five resistance values, indicated in Figure P3.11.

**Find:**
The current, $i$, drawn from the independent voltage source using superposition.

**Analysis:**
(1) Suppress voltage source $V$. Redraw the circuit.

At node 1:
\[ \frac{v_1 - v_1 + v_1 - v_2}{0.5 + 0.5 + 0.25} = 0 \]
At node 2:
\[ \frac{v_2 - v_1 + v_2 + 0.5}{0.25 + 0.75} = 0 \]
Solving the system, we obtain:
\[ v_1 = -0.075 \text{ V}, \quad v_2 = -0.15 \text{ V} \]
Therefore, \( i_1 = \frac{-v_1}{0.5} = 150 \text{ mA} \).

(2) Suppress current source $I$. Redraw the circuit.

At node 1:
\[ \frac{v_1 - \frac{3}{0.5} + v_1 + v_1}{0.5 + 0.5 + (0.25 + 0.5 + 0.25)} = 0 \]
Solving,
\[ v_1 = 1.2 \text{ V} \]
Therefore, \( i_1 = \frac{3 - v_1}{0.5} = 3.6 \text{ A} \).

Using the principle of superposition,
\[ I = i_1 + i_2 = 3.75 \text{ A} \]
Problem 3.48

Solution:

Known quantities:

\[ V_{S2} = kT \quad k = 10 \text{ V/°C} \]
\[ V_{S1} = 24 \text{ V} \quad R_S = R_1 = 12 \text{ kΩ} \]
\[ R_2 = 3 \text{ kΩ} \quad R_3 = 10 \text{ kΩ} \]
\[ R_4 = 24 \text{ kΩ} \quad V_{R3} = -2.524 \text{ V} \]

Circuit in Figure P3.23:

The voltage across \( R_3 \), which is given, indicates the temperature.

Find:
The temperature, \( T \) using superposition.

Analysis:

(1) Suppress voltage source \( V_{S2} \). Redraw the circuit.

For mesh (a):
\[ i_a(24k) + i_b(-12k) + i_c(-12k) = 24 \]
For mesh (b):
\[ i_a(-12k) + i_b(46k) + i_c(-10k) = 0 \]
For mesh (c):
\[ i_a(-12k) + i_b(-10k) + i_c(25k) = 0 \]
Solving,
\[ i_a = 2.08 \text{ mA}, \quad i_b = 0.83 \text{ mA} \text{ and } i_c = 1.33 \text{ mA}. \]

Therefore,
\[ V_{R3,S2} = 10000(i_b - i_c) = -5 \text{ V}. \]

(2) Suppress voltage source \( V_{S1} \). Redraw the circuit.

For mesh (a):
\[ i_a(24k) + i_b(-12k) + i_c(-12k) + 10T = 0 \]
For mesh (b):
\[ i_a(-12k) + i_b(46k) + i_c(-10k) = 10T \]
For mesh (c):
\[ i_a(-12k) + i_b(-10k) + i_c(25k) = 0 \]
Solving,
\[ i_a = -0.52T \text{ mA}, \quad i_b = 0.029T \text{ mA} \text{ and } i_c = -0.2381T \text{ mA}. \]

Therefore,
\[ V_{R3,S1} = 10000(i_b - i_c) = 2.671T \text{ V}. \]

Using the principle of superposition,
\[ V_{R3} = V_{R3,S2} + V_{R3,S1} = -5 + 2.671T = -2.524 \text{ V} \]
Therefore,
\[ T = 0.926 \text{ °C}. \]
Problem 3.49

Solution:

**Known quantities:**
The values of the resistors and of the voltage sources (see Figure P3.25).

**Find:**
The voltage across the $10 \, \Omega$ resistor in the circuit of Figure P3.14 using superposition.

**Analysis:**

(1) Suppress voltage source $V_{S1}$. Redraw the circuit.

For mesh (a):

\[ i_a (50 + 20 + 20) - i_b (20) - i_c (20) = 0 \]

For mesh (b):

\[ -i_a (20) + i_b (20 + 10) - i_c (10) + 5 = 0 \]

For mesh (c):

\[ -i_a (20) - i_b (10) + i_c (20 + 10 + 15) = 0 \]

Solving,

\[ i_a = -73.8 \, mA \] , \[ i_b = -245 \, mA \] and \[ i_c = -87.2 \, mA \].

Therefore,

\[ V_{10 \, \Omega, S1} = 10(i_b - i_c) = -1.578 \, V \].

(2) Suppress voltage source $V_{S2}$. Redraw the circuit.

For mesh (a):

\[ i_a (50 + 20 + 20) - i_b (20) - i_c (20) = 12 \]

For mesh (b):

\[ -i_a (20) + i_b (20 + 10) - i_c (10) = 0 \]

For mesh (c):

\[ -i_a (20) - i_b (10) + i_c (20 + 10 + 15) = 0 \]

Solving,

\[ i_a = 201 \, mA \] , \[ i_b = 177 \, mA \] and \[ i_c = 129 \, mA \].

Therefore,

\[ V_{10 \, \Omega, S1} = 10(i_b - i_c) = 0.48 \, V \].

Using the principle of superposition,

\[ V_{10 \, \Omega} = V_{10 \, \Omega, S2} + V_{10 \, \Omega, S1} = -1.09 \, V \].
Problem 3.50

Solution:

Known quantities:
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.26.

Find:
The voltage across the current source using superposition.

Analysis:
(1) Suppress voltage source. Redraw the circuit.
For mesh (a):
\[ i_a(20+30) + i_b(30) = 0 \]
For meshes (b) and (c):
\[ i_a(30) + i_b(10+30) + i_c(30+20) = 0 \]
For the current source:
\[ i_c - i_b = 0.5 \]
Solving,
\[ i_a = -208 \text{ mA}, \ i_b = -347 \text{ mA} \text{ and } i_c = 153 \text{ mA}. \]
Therefore,
\[ v_V = i_c(30+20) = 7.65 \text{ V}. \]

(2) Suppress current source. Redraw the circuit.
For mesh (a):
\[ i_a(20+30) + i_b(30) = 3 \]
For mesh (b):
\[ i_a(30) + i_b(90) = 0 \]
Solving,
\[ i_a = 75 \text{ mA} \text{ and } i_b = 25 \text{ mA}. \]
Therefore,
\[ v_I = i_b(30+20) = 1.25 \text{ V}. \]
Using the principle of superposition,
\[ v = v_V + v_I = 8.9 \text{ V}. \]
### Problem 3.51

**Solution:**

**Known quantities:**
Circuit shown in Figure P3.51.

**Find:**
Thevenin equivalent circuit

**Analysis:**
- \( R_{TH} = 1\Omega + 4\Omega = 5\Omega = 3.22\Omega \)
- \( R_{TH} = 1 + \frac{1}{4} = 1 + \frac{20}{9} = \frac{29}{9} = 3.22\Omega \)

Voltage divider gives
- \( V = \left( \frac{4}{4+5} \right) 36 = 16V \)

KVL:
- \( v_{dc} = -0(1) + v = v = 16V \)

### Problem 3.52

**Solution:**

**Known quantities:**
Circuit shown in Figure P3.52.

**Find:**
Thevenin equivalent circuit and the voltage across resistance 3Ω

**Analysis:**
- **KVL:** \( v_3 = v_1 + v_2 \)
- **Find \( v_{dc} \) (3 Ω disconnected)\)
- **KCL Node 1:**
  \[
  2 + \frac{1}{2} (v_1 - v_2) + \frac{1}{2} [v_1 - (3 + v_2)] = 0
  \]
- **KCL Nodes 2 & 3:**
  \[
  \frac{1}{2} (v_2 - v_1) + \frac{1}{4} v_2 + \frac{1}{2} (3 + v_2) - v_1 = 0
  \]
  \[
  1v_1 - 1v_2 = -\frac{1}{2} \\
  -1v_1 + \frac{5}{4} v_2 = -\frac{3}{2}
  \]
  \[
  \Rightarrow v_1 = -\frac{17}{2} = -8.5 \text{ V} \\
  v_2 = -8 \text{ V} \Rightarrow v_{TH} = v_3 = v_2 + 3 = -5 \text{ V}
  \]
- \( R_{TH} = 4\Omega \)
- \( v = \frac{3}{4+3} (-5) = -\frac{15}{7} V = -2.14 \text{ V} \)
Problem 3.53

Solution:

Known quantities:
Circuit shown in Figure P3.53.

Find:
Norton equivalent circuit

Analysis:

\[ R_N = 3\Omega + 1\Omega + \frac{3\Omega \cdot 1\Omega}{1\Omega} = 4.75\Omega \]

Using the mesh analysis approach

\[ 4i_1 - 3i_2 = 2 \]
\[ -3i_1 + 4i_2 + 3i_{sc} = 0 \]
\[ i_2 - i_{sc} = 2 \]

Solving, \( i_{sc} = -0.42A \Rightarrow i_N = -0.42 \text{ A} \)

It means the magnitude of \( i_{sc} \) is 0.42 A and the direction of \( i_{sc} \) is count-clockwise.

Problem 3.54

Solution:

Known quantities:
Circuit shown in Figure P3.54.

Find:
Norton equivalent circuit

Analysis:

\[ R_N = 5\Omega \parallel \left( 3\Omega + 2\Omega \parallel 1\Omega \right) = 2.12\Omega \]

Using mesh analysis,

\[ 8 - 1(i_1 - i_2) - 2(i_1 - i_{sc}) = 0 \]
\[ -1(i_2 - i_1) - 5i_2 - 3(i_2 - i_{sc}) = 0 \]
\[ -2(i_{sc} - i_1) - 3(i_{sc} - i_2) = 0 \]

Solving, \( i_{sc} = 3.05A \Rightarrow I_N = 3.05A \)
Problem 3.55

Solution:

Known quantities:
Circuit shown in Figure P3.55.

Find:
Thevenin equivalent circuit

Analysis:
To find $R_T$, we zero the two sources by shorting the voltage source and opening the current source. The resulting circuit is shown in the left:
Therefore, $R_T = 1,000 \parallel 1,000 + 1 + 3 = 504 \Omega$.

To find $V_{oc}$, we assume $V_b$ as reference (i.e., zero) and apply nodal analysis.

\[
\frac{V_c - V_a}{1} = 0.01
\]

\[
\frac{10 - V_c}{1000} = \frac{V_c}{1000} + 0.01
\]

Then, $V_a = -0.01V$

Therefore, $V_{OC} = -0.01V$.

It means that $a$ is negative side, $b$ is positive side while the magnitude of $V_{oc}$ is 0.01V.

Problem 3.56

Solution:

Known quantities:
Circuit shown in Figure P3.56.

Find:
Thevenin equivalent circuit

Analysis:
To find $R_T$, we need to make the current source an open circuit and the voltage sources short circuits, as follows:

Note that this circuit has only three nodes. Thus, we can re-draw the circuit as shown:

and combine the two parallel resistors to obtain:

Thus, $R_T = 50 \parallel (50 + 33.3) \parallel 100 = 23.81 \Omega$
Problem 3.57

Solution:

Known quantities:
Circuit shown in Figure P3.57.

Find:
Thevenin equivalent circuit

Analysis:
To find $R_T$,

Therefore,

$R_T = \{(8||8) + 2\||3\} + 8 = 10 \, \Omega$

To find $v_{OC}$, nodal analysis can be applied. Note that the 8 $\Omega$ resistor may be omitted because no current flows through it, and it therefore does not affect $v_{OC}$.

$$ \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{2}\right) v_1 - \frac{1}{2} v_{OC} = \frac{12}{8} $$
$$ -\frac{1}{2} v_1 + \left(\frac{1}{2} + \frac{1}{3}\right) v_{OC} = 0 $$

or

$3v_1 - 2v_{OC} = 6$
$-3v_1 + 5v_{OC} = 0$

Therefore, $v_{OC} = 2 \, V$
Problem 3.58

Solution:

Known quantities:
Circuit shown in Figure P3.58.

Find:
Thevenin equivalent circuit

Analysis:
To find $R_T$, we short circuit the source

Starting from the left side,

\[(1 + 0.1) \parallel 10 = 0.99 \Omega ,\]
\[(1 + 0.99 + 0.1) \parallel 20 = 1.893 \Omega \]

Therefore, we have

\[R_T = 1.893 + 0.1 + 1 = 2.993 \Omega .\]

To find $v_{OC}$, we apply mesh analysis:

Two resistors are omitted because no current flows through them and they, therefore, do not affect $v_{OC}$.

\[(1 + 0.1 + 10) i_1 - 10 i_2 = 15\]
\[(1 + 20 + 0.1 + 10) i_2 - 10 i_1 = 0\]

Solving for $i_2$,

\[i_2 = 0.612 \text{ A}\]

we obtain,

\[v_T = v_{OC} = 20 i_2 = 12.24 \text{ V}\]

Problem 3.59

Solution:

Known quantities:
Circuit shown in Figure P3.59.

Find:
Value of resistance $R_x$

Analysis:

a) We have

\[V_{ab} = V_a - V_b = \frac{R}{R + R} V_S - \frac{R_x}{R + R_x} V_S\]

\[V_{ab} = \frac{1}{2} V_S - \frac{R_x}{R + R} V_S\]

b) For $R = 1 \text{ kW}$, $V_S = 12 \text{ V}$, $V_{ab} = 12 \text{ mV}$,

\[0.012 = 6 \cdot \frac{R_x}{1000 + R_x} - 12\]

\[R_x = 996 \Omega\]
Problem 3.60

Solution:

Known quantities:
Circuit shown in Figure P3.60.

Find:

a) Thevenin equivalent resistance
b) Power dissipated by \( R_L \)
c) Power dissipated by \( R_T \) and \( R_L \)
d) Power dissipated by the bridge without the load resistor

Analysis:

To find \( R_T \), short circuit \( v_S \). Thus,

\[
R_T = (R_1 || R_2) + (R_3 || R_4) = 999 \, \Omega
\]

\( v_T = v_S \cdot \frac{R_x}{R + R_x} v_S = 12 \, \text{mV} \)

b) Using the circuit shown:

\[
P_{300\Omega} = \frac{v^2}{R} = 32.04 \times 10^{-9} = 32 \, \text{nW}
\]

c) Using the previous circuit,

\[
P_{RT} = \frac{v^2}{R_T} = 64 \, \text{nW}
\]

d) With no load resistor, \( P_{dissipated} = \frac{12^2}{2000} + \frac{12^2}{1996} = 144.1 \, \text{mW} \)

Problem 3.61

Note: the dependent current source on the left should be labeled \( i_1 \). Further, assume that the two voltage sources do not source any current.

Solution:

Known quantities:
Circuit shown in Figure P3.61.

Find:

\( v_o \) as an expression of \( v_1 \) and \( v_2 \).

Analysis:

Taking the bottom node as the reference,

\[
v_{O-} = -4i_1, \quad v_{O+} = -4i_2
\]

Then,

\[
v_O = v_{O-} - v_{O+} = -4i_2 + 4i_1 = 4(i_1 - i_2)
\]

But,

\[
i_1 = \frac{1}{2}[v_1 - 5(i_1 + i_2)]
\]

\[
i_2 = \frac{1}{2}[v_2 - 5(i_1 + i_2)]
\]

So,

\[
v_O = 2(v_1 - v_2)
\]

Note: \( v_1 \) and \( v_2 \) do not source any current.
**Problem 3.62**

**Solution:**

**Known quantities:**
The schematic of the circuit (see Figure P3.5).

**Find:**
The Thévenin equivalent resistance seen by resistor \( R_3 \), the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when \( R_3 \) is the load.

**Analysis:**

1. Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

\[
R_T = R_1 \parallel R_4 + R_2 \parallel R_3 = \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_3}{R_2 + R_3}
\]

2. Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:

\[
\frac{v_1}{R_1} + \frac{v_1 - V_{S1}}{R_4} = 0
\]

For node #2:

\[
\frac{v_2}{R_2} + \frac{v_2 + V_{S2}}{R_5} = 0
\]

Solving the system,

\[
v_1 = -\frac{R_1}{R_1 + R_4} V_{S1}
\]

\[
v_2 = -\frac{R_2}{R_2 + R_5} V_{S2}
\]

Therefore,

\[
V_{OC} = v_1 - v_2 = \frac{R_1}{R_1 + R_4} V_{S1} + \frac{R_2}{R_2 + R_5} V_{S2}
\]

3. Replace the load with a short circuit. Redraw the circuit.

For mesh (a): \( i_a (R_1 + R_4) - R_1 i_c = V_{S1} \)

For mesh (b): \( i_b (R_2 + R_5) - R_2 i_c = V_{S2} \)

For mesh (c): \( -R_1 i_a - R_2 i_b + i_c (R_1 + R_2) = 0 \)

Solving the system,

\[
i_a = \frac{(R_1 R_2 + R_1 R_5 + R_2 R_3) V_{S1} + R_1 R_2 V_{S2}}{R_1 R_4 (R_2 + R_5) + R_2 R_5 (R_1 + R_4)}
\]

\[
i_b = \frac{R_1 R_2 V_{S1} + (R_1 R_2 + R_1 R_4 + R_2 R_4) V_{S2}}{R_1 R_4 (R_2 + R_5) + R_2 R_5 (R_1 + R_4)}
\]

\[
i_c = \frac{R_1 (R_2 + R_5) V_{S1} + R_2 (R_1 + R_4) V_{S2}}{R_1 R_4 (R_2 + R_5) + R_2 R_5 (R_1 + R_4)}
\]

Therefore,

\[
i_{SC} = i_c = \frac{R_1 (R_2 + R_5) V_{S1} + R_2 (R_1 + R_4) V_{S2}}{R_1 R_4 (R_2 + R_5) + R_2 R_5 (R_1 + R_4)}
\]
Problem 3.63

Solution:

Known quantities:
The schematic of the circuit (see Figure P3.10).

Find:
The Thévenin equivalent resistance seen by resistor $R_L$, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when $R_L$ is the load.

Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 25 \, \Omega \parallel (75 \, \Omega + 200 \, \Omega) = 22.92 \, \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:

$$\frac{V_1}{200} + \frac{V_1 - V_2}{75} = 0.2$$

For node #2:

$$\frac{V_2 - V_1}{75} + \frac{V_2 - V_3}{25} + \frac{V_2 - V_3}{50} + i_{10V} = 0$$

For node #3:

$$\frac{V_3 - V_2}{50} = i_{10V}$$

For the voltage source:

$$V_3 + 10 = V_2$$

Solving the system,

$$V_1 = 13.33 \, \text{V}, \quad V_2 = 3.33 \, \text{V} \quad \text{and} \quad V_3 = -6.67 \, \text{V}.$$  

Therefore,

$$v_{oc} = V_3 = -6.67 \, \text{V}.$$  

(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a):

$$i_a(50) = 10$$

For mesh (b):

$$i_b(300) - i_c(25) = 40$$

For mesh (c):

$$i_b(25) - i_c(25) = 10$$

Solving the system,

$$i_a = 200 \, \text{mA}, \quad i_b = 109 \, \text{mA} \quad \text{and} \quad i_c = -291 \, \text{mA}.$$  

Therefore,

$$i_{sc} = i_c = -291 \, \text{mA}.$$
Problem 3.64

Solution:

Known quantities:
The schematic of the circuit (see Figure P3.23).

Find:
The Thévenin equivalent resistance seen by resistor $R_5$, the
Thévenin (open-circuit) voltage and the Norton (short-circuit)
current when $R_5$ is the load.

Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 0.5 \, \Omega + 0.25 \, \Omega + (0.5 \, \Omega \parallel 0.5 \, \Omega) = 1 \, \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:
$$\frac{v_1 - 3}{0.5} + \frac{v_1}{0.5} + \frac{v_1 - v_2}{0.25} = 0$$

For node #2:
$$\frac{v_2 - v_1}{0.25} + 0.5 = 0$$

Solving the system,
$v_1 = 1.375 \, V$ and $v_2 = 1.25 \, V$.

Therefore,
$v_{OC} = v_2 = 1.25 \, V$.

(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a):
$$i_a (0.5 + 0.5) - i_b (0.5) = 3$$

For meshes (b) and (c):
$$-i_a (0.5) + i_b (0.5 + 0.25) + i_c (0.5) = 0$$

For the current source:
$$i_b - i_c = 0.5$$

Solving the system,
$$i_a = 3.875 \, A \text{, } i_b = 1.75 \, A \text{ and } i_c = 1.25 \, A.$$  

Therefore,
$$i_{SC} = i_c = 1.25 \, A.$$
Problem 3.65

Solution:

Known quantities:
The schematic of the circuit (see Figure P3.23).

Find:
The Thévenin equivalent resistance seen by resistor $R_3$, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when $R_3$ is the load.

Assumption:
As in P3.12, we assume $T = 0.926^\circ$C, so that $V_{S2} = 9.26$ V.

Analysis:

1. Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

\[ R_T = 12 \, \text{kΩ} \parallel 12 \, \text{kΩ} + 3 \, \text{kΩ} \parallel 24 \, \text{kΩ} = 8.67 \, \text{kΩ} \]

2. Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:
\[ \frac{v_1 - 24}{12000} + \frac{v_1 - 9.26}{12000} = 0 \]
For node #2:
\[ \frac{v_2 - 24}{3000} + \frac{v_2}{24000} = 0 \]
Solving the system, $v_1 = 16.63$ V and $v_2 = 21.33$ V.
Therefore, $v_{OC} = v_1 - v_2 = -4.7$ V.

3. Replace the load with a short circuit. Redraw the circuit.

For mesh (a):
\[ i_a(24k) + i_b(12k) - i_c(12k) = 24 - 9.26 \]
For mesh (b):
\[ -i_a(12k) + i_b(36k) = 9.26 \]
For mesh (c):
\[ -i_a(12k) + i_c(15k) = 0 \]
Solving the system, $i_a = 1.71$ mA, $i_b = 0.83$ mA and $i_c = 1.37$ mA.
Therefore, $i_{SC} = i_b - i_c = -0.54$ mA.
Problem 3.66

Solution:

Known quantities:
The schematic of the circuit (see Figure P3.25).

Find:
The Thévenin equivalent resistance seen by resistor $R_4$, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when $R_4$ is the load.

Analysis:

1. Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

\[ R_T = R_2 \parallel \left( R_3 + (R_1 \parallel R_5) \right) = 20 \Omega \parallel \left( 20 \Omega + (50 \Omega \parallel 15 \Omega) \right) = 12.24 \Omega \]

2. Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:

\[ \frac{v_1 - 12}{50} + \frac{v_1 - v_2}{20} + i_{SV} = 0 \]

For node #2:

\[ \frac{v_2 - v_1}{20} + \frac{v_2}{20} = 0 \]

For node #3:

\[ \frac{v_3}{15} - i_{SV} = 0 \]

For the 5-V voltage source:

\[ v_1 - v_3 = 5 \]

Solving the system,

\[ v_1 = 5.14 \text{ V}, \quad v_2 = 2.57 \text{ V}, \quad v_1 = 0.13 \text{ V} \text{ and } i_{SV} = 8.95 \text{ mA} \]

Therefore,

\[ v_{OC} = v_2 - v_3 = 2.44 \text{ V} \]

3. Replace the load with a short circuit. Redraw the circuit.

For mesh (a):

\[ i_a(90) - i_b(20) - i_c(20) = 12 \]

For mesh (b):

\[ -i_a(20) + i_b(20) + 5 = 0 \]

For mesh (c):

\[ -i_a(20) + i_c(35) = 0 \]

Solving the system,

\[ i_a = 119.5 \text{ mA}, \quad i_b = -130.5 \text{ mA} \text{ and } i_c = 68.3 \text{ mA} \]

Therefore,

\[ i_{SC} = i_c - i_b = 198.8 \text{ mA} \]
Problem 3.67

Solution:

Known quantities:
The schematic of the circuit (see Figure P3.26).

Find:
The Thévenin equivalent resistance seen by resistor $R_s$, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when $R_s$ is the load.

Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 30 \, \Omega + 10 \, \Omega + (20 \, \Omega || 30 \, \Omega) = 52 \, \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:

$$\frac{v_1 - 3}{20} + \frac{v_1}{30} + \frac{v_1 - v_2}{10} = 0$$

For node #2:

$$\frac{v_2 - v_1}{10} = 0.5$$

Solving the system,

$$v_1 = 7.8 \, \text{V} \quad \text{and} \quad v_2 = 12.8 \, \text{V}$$

Therefore,

$$v_{OC} = v_2 = 12.8 \, \text{V}$$

(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a):

$$i_a (20 + 30) - i_b (30) = 3$$

For meshes (b) and (c):

$$-i_a (30) + i_b (30 + 10) + i_c (30) = 0$$

For the current source:

$$i_c - i_b = 0.5$$

Solving the system,

$$i_a = -92 \, \text{mA}, \quad i_b = -254 \, \text{mA} \quad \text{and} \quad i_c = 246 \, \text{mA}$$

Therefore,

$$i_{SC} = i_c = 246 \, \text{mA}$$
Problem 3.68

Solution:

Known quantities:
The schematic of the circuit (see Figure P3.41).

Find:
The Thévenin equivalent resistance seen by resistor $R$, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when $R$ is the load.

Analysis:
(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 1 \, \Omega \parallel 0.3 \, \Omega = 0.23 \, \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1: $$\frac{v_1}{1} + \frac{v_1 - 12}{0.3} = 12$$
Solving, $$v_1 = 12 \, V$$.
Therefore, $$v_{OC} = v_1 = 12 \, V$$.

(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a): $$i_a (1 + 0.3) - i_b (0.3) - 12(1) + 12 = 0$$
For mesh (b): $$-i_a (0.3) + i_b (0.3) = 12$$
Solving the system, $$i_a = 12 \, A$$ and $$i_b = 52 \, A$$.
Therefore, $$i_{SC} = i_b = 52 \, A$$.
Problem 3.69

Solution:

Known quantities:
The schematic of the circuit (see Figure P3.43).

Find:
The Thévenin equivalent resistance seen by resistor $R_3$, the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when $R_3$ is the load.

Analysis:
(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit.

$$R_T = 1 \Omega || 7 \Omega + 1 \Omega || 5 \Omega = 1.71 \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit.

For node #1:

$$\frac{v_1 - 450}{1} + \frac{v_1}{7} = 0$$

For node #2:

$$\frac{v_2 + 450}{1} + \frac{v_2}{5} = 0$$

Solving the system, $v_1 = 393.75 \text{ V}$ and $v_2 = -375 \text{ V}$.

Therefore, $v_{OC} = v_1 - v_2 = 768.75 \text{ V}$.

(3) Replace the load with a short circuit. Redraw the circuit.

For mesh (a):

$$i_a(1 + 7) - i_c(7) = 450$$

For mesh (b):

$$i_b(5 + 1) - i_c(5) = 450$$

For mesh (c):

$$-i_a(7) - i_b(5) + i_c(7 + 5) = 0$$

Solving the system, $i_a = 450 \text{ A}$, $i_b = 450 \text{ A}$ and $i_c = 450 \text{ A}$.

Therefore, $i_{SC} = i_c = 450 \text{ A}$.
**Problem 3.70**

**Solution:**

**Known quantities:**
The values of the voltage source, \( V_S = 110 \text{ V} \), and the values of the 4 resistors in the circuit of Figure P3.70:

\[
R_1 = R_2 = 930 \text{ m}\Omega \quad R_3 = 100 \text{ m}\Omega \quad R_S = 19 \text{ m}\Omega
\]

**Find:**
The change in the voltage across the total load, when the customer connects the third load \( R_3 \) in parallel with the other two loads.

**Analysis:**
Choose a ground. If the node at the bottom is chosen as ground (which grounds one terminal of the ideal source), the only unknown node voltage is the required voltage. Specify directions of the currents and polarities of voltages.

Without \( R_3 \):

**KCL:**

\[
I_S + I_1 + I_2 = 0
\]

**OL:**

\[
\frac{V_{RS}}{R_S} + \frac{V_{R1}}{R_1} + \frac{V_{R2}}{R_2} = 0
\]

\[
\frac{V_{Oi} - V_S}{R_S} + \frac{V_{Oi} - 0}{R_1} + \frac{V_{Oi} - 0}{R_2} = 0
\]

\[
V_{Oi} = \frac{V_S}{R_S + \frac{1}{R_1} + \frac{1}{R_2}} = \frac{110}{1.04086} = 105.7 \text{ V}
\]

With \( R_3 \):

**KCL:**

\[
I_S + I_1 + I_2 + I_3 = 0
\]

**OL:**

\[
\frac{V_{RS}}{R_S} + \frac{V_{R1}}{R_1} + \frac{V_{R2}}{R_2} + \frac{V_{R3}}{R_3} = 0
\]

\[
\frac{V_{Of} - V_S}{R_S} + \frac{V_{Of} - 0}{R_1} + \frac{V_{Of} - 0}{R_2} + \frac{V_{Of} - 0}{R_3} = 0
\]

\[
V_{Of} = \frac{V_S}{R_S + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{110}{1.04086 + 0.19} = 89.37 \text{ V}
\]

Therefore, the voltage decreased by:

\[
\Delta V_o = V_{Of} - V_{Oi} = -16.33 \text{ V}
\]

**Notes:**
1. "Load" to an EE usually means current rather than resistance.
2. Additional load reduces the voltage supplied to the customer because of the additional voltage dropped across the losses in the distribution system.
Problem 3.71

Solution:

Known quantities:
The values of the voltage source, $V_S = 450$ V, and the values of the 4 resistors in the circuit of Figure P3.71.

$$R_1 = R_2 = 1.3 \, \Omega \quad R_3 = 500 \, \text{m}\Omega \quad R_S = 19 \, \text{m}\Omega$$

Find:
The change in the voltage across the total load, when the customer connects the third load $R_3$ in parallel with the other two loads.

Analysis:
See Solution to Problem 3.70 for a detailed mathematical analysis.

Problem 3.72

Solution:

Known quantities:
The circuit shown in Figure P3.72, the values of the terminal voltage, $V_T$, before and after the application of the load, respectively $V_T = 20$ V and $V_T = 18$ V, and the value of the load resistor $R_L = 2.7 \, \text{k}\Omega$.

Find:
The internal resistance and the voltage of the ideal source.

Analysis:

\[ KVL: \quad -V_S + I_T R_S + V_T = 0 \]

If \( I_T = 0 \):
\( V_S = V_T = 20 \, \text{V} \)

If \( V_T = 18 \, \text{V} \):
\[ I_T = \frac{V_T}{R_L} = 6.67 \, \text{mA} \quad \text{and} \quad R_S = \frac{V_S - V_T}{I_T} = 300 \, \Omega \]

Note that $R_S$ is an equivalent resistance, representing the various internal losses of the source and is not physically a separate component. $V_S$ is the voltage generated by some internal process. The source voltage can be measured directly by reducing the current supplied by the source to zero, i.e., no-load or open-circuit conditions. The source resistance cannot be directly measured; however, it can be determined, as was done above, using the interaction of the source with an external load.
Section 3.7: Maximum power transfer

Problem 3.73

**Solution:**

**Known quantities:**
The values of the voltage and of the resistor in the equivalent circuit of Figure P3.73: $V_{TH} = 12 \, \text{V}; \, R_{eq} = 8 \, \Omega$

**Assumptions:**
Assume the conditions for maximum power transfer exist.

**Find:**
- The value of $R_L$.
- The power developed in $R_L$.
- The efficiency of the circuit, that is the ratio of power absorbed by the load to power supplied by the source.

**Analysis:**
- For maximum power transfer: $R_L = R_{eq} = 8 \, \Omega$
- $V_D$:
  $$V_{RL} = \frac{V_{TH} R_L}{R_{eq} + R_L} = \frac{(12)(8)}{8 + 8} = 6 \, \text{V}$$
  $$P_{RL} = \frac{V_{RL}^2}{R_L} = \frac{(6)^2}{8} = 4.5 \, \text{W}$$
- Efficiency:
  $$\eta = \frac{P_L}{P_S} = \frac{P_{RL}}{P_{Req} + P_{RL}} = \frac{I_L^2 R_L}{I_{eq}^2 R_{eq} + I_L^2 R_L} = \frac{R_L}{R_{eq} + R_L} = 0.5 = 50\%$$

---

Problem 3.74

**Solution:**

**Known quantities:**
The values of the voltage and of the resistor in the equivalent circuit of Figure P3.73: $V_{TH} = 300 \, \text{V}; \, R_{eq} = 600 \, \Omega$

**Assumptions:**
Assume the conditions for maximum power transfer exist.

**Find:**
- The value of $R_L$.
- The power developed in $R_L$.
- The efficiency of the circuit, that is the ratio of power absorbed by the load to power supplied by the source.

**Analysis:**
- For maximum power transfer: $R_L = R_{eq} = 600 \, \Omega$
- $V_D$:
  $$V_{RL} = \frac{V_{TH} R_L}{R_{eq} + R_L} = \frac{(35)(600)}{600 + 600} = 17.5 \, \text{V}$$
  $$P_{RL} = \frac{V_{RL}^2}{R_L} = \frac{(17.5)^2}{600} = 510.4 \, \text{mW}$$
- Efficiency:
  $$\eta = \frac{P_L}{P_S} = \frac{P_{RL}}{P_{Req} + P_{RL}} = \frac{I_L^2 R_L}{I_{eq}^2 R_{eq} + I_L^2 R_L} = \frac{R_L}{R_{eq} + R_L} = 0.5 = 50\%$$
**Problem 3.75**

**Solution:**

**Known quantities:**
The values of the voltage source, \( V_S = 12 \text{ V} \), and of the resistance representing the internal losses of the source, \( R_S = 0.3 \Omega \), in the circuit of Figure P3.59.

**Find:**
a. Plot the power dissipated in the load as a function of the load resistance. What can you conclude from your plot?
b. Prove, analytically, that your conclusion is valid in all cases.

**Analysis:**

\[ -V_S = IR_S + IR = 0 \]

\( R \ [\Omega] \quad I \ [\text{A}] \quad P_R \ [\text{W}] \)

\begin{array}{|c|c|c|}
\hline
R & I & P_R \\
\hline
0 & 40 & 0.0 \\
0.1 & 30 & 90.0 \\
0.3 & 20 & 120.0 \\
0.9 & 10 & 90.0 \\
2.1 & 5 & 52.5 \\
\hline
\end{array}

b. \[ P_R = I^2R = \frac{V_S^2R}{(R + R_S)^2} = V_S^2R(R + R_S)^{-2} \]

\[ \frac{dP_R}{dR} = V_S^2(1)(R + R_S)^{-2} + V_S^2(R)(-2)(R + R_S)^{-3}(1) = 0 \]

\[ (R + R_S)^{-1} - 2R = 0 \quad \Rightarrow \quad R = R_S \]
Section 3.8: Nonlinear circuit elements

Problem 3.76

Solution:

Known quantities:
The two nonlinear resistors, in the circuit of Figure P3.76, are characterized by:
\[ i_a = 2v_a^3 \quad i_b = v_b^3 + 10v_b \]

Find:
The node voltage equations in terms of \( v_1 \) and \( v_2 \).

Analysis:
At node 1, \[ \frac{v_1}{1} + i_a = 1 \Rightarrow v_1 + 2v_a^3 = 1 \]
At node 2, \[ i_b - i_a = 26 \Rightarrow v_b^3 + 10v_b - 2v_a^3 = 26 \]
But \( v_a = v_1 - v_2 \) and \( v_b = v_2 \). Therefore, the node equations are
\[ v_1 + 2(v_1 - v_2)^3 = 1 \quad \text{and} \quad v_2^3 + 10v_2 - 2(v_1 - v_2)^3 = 26 \]

Problem 3.77

Solution:

Known quantities:
The characteristic \( I-V \) curve shown in Figure P3.77, and the values of the voltage, \( V_T = 15 \text{ V} \), and of the resistance, \( R_T = 200 \Omega \), in the circuit of Figure P3.77.

Find:
a. The operating point of the element that has the characteristic curve shown in Figure P3.61.
b. The incremental resistance of the nonlinear element at the operating point of part a.
c. If \( V_T \) were increased to 20 V, find the new operating point and the new incremental resistance.

Analysis:

a. \( KVL: \)
\[ -15 + 200I + V = 0 \]
Solving for \( V \) and \( I \),
\[ I = 52.2 \text{ mA} \quad V = 4.57 \text{ V} \quad \text{or} \quad -6.57 \text{ V} \]
The second voltage value is physically impossible.

b. \( R_{\text{inc}} = 10(0.0522)^{-0.5} = 43.8 \Omega \)

c. \( I = 73 \text{ mA} \quad V = 5.40 \text{ V} \quad R_{\text{inc}} = 37 \Omega \)

Problem 3.78

Solution:

Known quantities:

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. Limited distribution permitted only to teachers and educators for course preparation. If you are a student using this Manual, you are using it without permission.
The characteristic \( I-V \) curve shown in Figure P3.78, and the values of the voltage, \( V_S = 450 \) V, and of the resistance, \( R = 9 \Omega \), in the circuit of Figure P3.78.

**Find:**
The current through and the voltage across the nonlinear device.

**Analysis:**
The \( I-V \) characteristic for the nonlinear device is given. Plot the circuit \( I-V \) characteristic, i.e., the DC load line.

\[
\text{KVL:} \quad -V_S + I_D R + V_D = 0
\]

\[
I_D = \frac{V_S - V_D}{R} = \frac{450 - V_D}{9}
\]

- \( I_D = 0 \) A if \( V_D = 450 \) V
- \( I_D = 50 \) A if \( V_D = 0 \)

The DC load line [circuit characteristic] is linear. Plotting the two intercepts above and connecting them with a straight line gives the DC load line. The solution for \( V \) and \( I \) is at the intersection of the device and circuit characteristics:

\( I_{DQ} = 26 \) A \( V_{DQ} = 210 \) V.

---

**Problem 3.79**

**Solution:**

**Known quantities:**
The \( I-V \) characteristic shown in Figure P3.79, and the values of the voltage, \( V_S = V_T = 1.5 \) V, and of the resistance, \( R = R_{eq} = 60 \) \( \Omega \), in the circuit of Figure P3.63.

**Find:**
The current through and the voltage across the nonlinear device.

**Analysis:**
The solution is at the intersection of the device and circuit characteristics. The device \( I-V \) characteristic is given. Determine and plot the circuit \( I-V \) characteristic.

\[
\text{KVL:} \quad -V_S + I_D R + V_D = 0
\]

\[
I_D = \frac{V_S - V_D}{R} = \frac{1.5 \text{ V} - V_D}{60 \text{ } \Omega}
\]

- \( I_D = 0 \) A if \( V_D = 1.5 \) V
- \( I_D = 25 \) mA if \( V_D = 0 \)

The DC load line [circuit characteristic] is linear. Plotting the two intercepts above and connecting them with a straight line gives the DC load line. The solution is at the intersection of the device and circuit characteristics, or "Quiescent", or "Q", or "DC operating" point:

\( I_{DQ} = 12 \) mA \( V_{DQ} = 0.77 \) V.

---

**Problem 3.80**

**Solution:**

**Known quantities:**
The \( I-V \) characteristic shown in Figure P3.80 as a function of pressure.

**PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. Limited use by educators for course preparation. If you are a student using this Manual, you are using...**
Find: The DC load line, the voltage across the device as a function of pressure, and the current through the nonlinear device when \( p = 30 \text{ psig} \).

Analysis:
Circuit characteristic [DC load line]:

\[
\text{KVL: } -V_S + I_D R + V_D = 0
\]

\[
I_D = \frac{V_S - V_D}{R} = \frac{2.5 \text{ V} - V_D}{125 \Omega} = 0 \text{ A} \quad \text{if} \quad V_D = 2.5 \text{ V}
\]

\[
= 20 \text{ mA} \quad \text{if} \quad V_D = 0
\]

The circuit characteristic is a linear relation. Plot the two intercepts and connect with a straight line to plot the DC load line. Solutions are at the intersections of the circuit with the device characteristics, i.e.:

<table>
<thead>
<tr>
<th>( p ) [psig]</th>
<th>( V_D [V] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.14</td>
</tr>
<tr>
<td>20</td>
<td>1.43</td>
</tr>
<tr>
<td>25</td>
<td>1.18</td>
</tr>
<tr>
<td>30</td>
<td>0.91</td>
</tr>
<tr>
<td>40</td>
<td>0.60</td>
</tr>
</tbody>
</table>

The function is nonlinear. At \( p = 30 \text{ psig} \):

\[ V_D = 1.08 \text{ V} \quad I_D = 12.5 \text{ mA} \]

---

**Problem 3.81**

**Solution:**

**Known quantities:**

The \( I-V \) characteristic of the nonlinear device in the circuit shown in Figure P3.81:

\[ I_D = I_0 e^{V_D/V_T} \]

\[ I_0 = 10^{-15} \text{ A} \quad V_T = 26 \text{ mV} \]

\[ V_S = V_T = 1.5 \text{ V} \]

\[ R = R_{eq} = 60 \Omega \]

Find: An expression for the DC load line. The voltage across and current through the nonlinear device.

**Analysis:**

Circuit characteristic [DC load line]:

\[
\text{KVL: } -V_S + I_D R + V_D = 0
\]

\[
[1] \quad I_D = \frac{V_S - V_D}{R} = \frac{1.5 - V_D}{60}
\]

\[
[2] \quad V_D = V_T \ln \left( \frac{I_D}{I_0} \right) = 0.026 \cdot \ln \left( \frac{I_D}{10^{-15}} \right)
\]

Iterative procedure:

Initially guess \( V_D = 750 \text{ mV} \). Note this voltage must be between zero and the value of the source voltage.

Then:

a. Use Equation [1] to compute a new \( I_D \).

b. Use Equation [2] to compute a new \( V_D \).
Problem 3.82

Solution:

Known quantities:
The $I$-$V$ characteristic shown in Figure P3.82 as a function of pressure.

- $V_S - V_T = 2.5$ V
- $R - R_{eq} = 125 \, \Omega$

Find:
The DC load line, and the current through the nonlinear device when $p = 40$ psig.

Analysis:

Circuit characteristic [DC load line]:

KVL: $-V_S + I_D R + V_D = 0$

$$I_D = \frac{V_S - V_D}{R} = \frac{2.5 \, \text{V} - V_D}{125 \, \Omega}$$

- $0$ A if $V_D = 2.5$ V
- $20$ mA if $V_D = 0$

The circuit characteristic is a linear relation that can be plotted by plotting the two intercepts and connecting them with a straight line. Solutions are at the intersections of the circuit and device characteristics.

At $p = 40$ psig: $V_D = 0.60$ V $I_D = 15.2$ mA

---

<table>
<thead>
<tr>
<th>$V_D$ [mV]</th>
<th>$I_D$ [mA]</th>
</tr>
</thead>
<tbody>
<tr>
<td>750</td>
<td>12.5</td>
</tr>
<tr>
<td>784.1</td>
<td>11.93</td>
</tr>
<tr>
<td>782.9</td>
<td>11.95</td>
</tr>
<tr>
<td>782.9</td>
<td>11.95</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

$I_{DQ} = 11.95$ mA $V_{DQ} = 782.9$ mV.
Problem 3.83

Solution:

Known quantities:
Circuit shown in Figure P3.83 and the program flowchart

Find:
   a) Graphical analysis of diode current and diode voltage.

Analysis:
   a) For every voltage value for the diode, we can calculate the corresponding current. Therefore we can calculate the voltage in the whole circuit. The intersection of voltage circuit and the thevenin equivalent voltage shows the answer.
      \[ v_D = 0.65 \]
      \[ i_D = 0.5 \]
   b) Run the attached Matlab code, we can have the following answer. They are close to the answer we got above.
      \[ v_D = 0.64 \]
      \[ i_D = 0.52 \]

```matlab
clear;close all;
ISAT=10e-12;kTq=0.0259;VT=12;RT=22;
VD1=VT/2;
VD2=VT;
flag=1;
while flag
    iD1=(VT-VD1)/RT;
    iD2=ISAT*(exp(VD1/kTq)-1);
    if iD1<iD2
        VD1=VD1+(VD2-VD1)/2;
    else
        VD2=VD1;
        VD1=VD1/2;
    end
    if abs(VD2-VD1)<10E-6;
        flag=0;
    end
end
iD1
VD1
```